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Studies on the real primitive infinite Lie algebras

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Introduction

By the classification theorem of Morimoto-Tanaka [1], we know that there are fourteen classes of the real primitive infinite Lie algebras. They contain all *the classical infinite Lie algebras* (see § 1). In the previous paper [2], we determined the structure of Lie subalgebras of finite codimension of *the classical infinite Lie algebras*.

One of the purposes of the present paper is to extend our result to the case of Lie subalgebras of finite codimension of the real primitive infinite Lie algebras. The precise statement can be seen in Theorem A.

By using Theorem A, we prove that fourteen classes of the real primitive infinite Lie algebras are not isomorphic to one another (see Theorem B).

§ 1. The real primitive infinite Lie algebras

Let V be an n-dimensional vector space over the field F, where F is $\mathbf{R}$ or $\mathbf{C}$. We denote by $L_{gl}(n, F)$ the Lie algebra of all formal vector fields over V. If $F=\mathbf{C}$, Lie subalgebras of $L_{gl}(n, \mathbf{C})$ are regarded as those of $L_{gl}(2n, \mathbf{R})$, and these "real" Lie algebras are denoted by the same notations.

The complete list of the real primitive infinite Lie algebras is following.

(1) $L_{gl}(n, \mathbf{R})$.

(2) $L_{sl}(n, \mathbf{R})$: the Lie algebra of real vector fields of divergence zero.

(3) $L_{csl}(n, \mathbf{R})$: the Lie algebra of real vector fields of constant divergence.

(4) $L_{sp}(2n, \mathbf{R})$: the Lie algebra of real Hamiltonian vector fields, $(n\geqq 2)$.

(5) $L_{csp}(2n, \mathbf{R})$: the Lie algebra of real vector fields preserving a Hamiltonian form up to a constant multiple, $(n\geqq 2)$.

(6) $L_{ct}(2n+1, \mathbf{R})$: the real contact algebra.

(7) $L_{gl}(n, \mathbf{C})$.

(8) $L_{sl}(n, \mathbf{C})$: the Lie algebra of complex vector fields of divergence zero.

(9) $L_{rsl}(n, \mathbf{C})$: the Lie algebra of complex vector fields with divergence on some real line in $\mathbf{C}$.