J. Math. Kyoto Univ. (JMKYAZ)
20-3 (1980) 435-446

A simple expression of the characters of certain discrete series representations, II

By

Shunsuke MIKAMI

(Communicated by Prof. H. Yoshizawa, april 2, 1979)

Introduction

In the previous paper [5], we showed that Hirai's general formula of the characters is reduced to a simple one for certain discrete series unitary representations of $SO_0(p, q)$ ($p+q$: odd). In this paper, we study the similar problem for the connected simple Lie group G of type FI, which is the unique exceptional Lie group of "class II".

Let $\mathfrak{g}$ be the Lie algebra of G, and B a compact Cartan subgroup of G. We denote by $\mathfrak{b}_c^*$ the complexification of the dual space of $\mathfrak{b}$, the Lie algebra of B, and $\mathfrak{b}_B^*$ the lattice in $\mathfrak{b}_c^*$ consisting of such $\Lambda \in \mathfrak{b}_c^*$ that the mapping $\xi_\Lambda : B \ni \exp X \longmapsto e^{\Lambda(X)}$ ($X \in \mathfrak{b}$) defines a unitary character of B. Let G$'$ be the totality of regular elements of G. Then by Harish-Chandra, it was shown that for each regular $\Lambda \in \mathfrak{b}_B^*$, there exists a discrete series representation of G whose character π_Λ is expressed on $B \cap G'$ as follows:

$$\pi_\Lambda = \varepsilon(\Lambda)\Big(\sum_{w \in W_k} \mathrm{sgn}(w)\xi_{w\Lambda}\Big)/\Delta^{\mathfrak{b}},$$

where $W_k(=W_G(\mathfrak{b}))$ denotes the little Weyl group and the number $\varepsilon(\Lambda) = \pm 1$ is determined by Λ and the positive system of the roots. In [1], Hirai gave a global formula of π_Λ' (the analytic function on G' corresponding to π_Λ) valid for any $\Lambda \in \mathfrak{b}_B^*$.

Since the root system of type F_4 belongs to "class II", Hirai's formula for type FI is very complicated for general Λ. In this note just as for the case of $SO_0(p, q)$ ($p+q$: odd), we study how the terms in the original formula are cancelling out each other when a regular element Λ in $\mathfrak{b}_B^*$ is dominant with respect to the positive system of Borel-de Siebenthal. We consider π_Λ' only on the connected component A of the identity