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An interacting system in population genetics, II

By

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1. Introduction

In the previous paper [10] we studied an interacting system in population genetics, which is called a continuous time stepping stone model. Let us review our model. Let S be a countable set. Each element i of S is called a colony. Assuming that there are two alleles A and B at each colony, we denote by x_i $(1-x_i)$ the gene frequency of the A-allele (resp. the B-allele) for the colony $i\in S$. We consider a time evolution of gene frequencies, which is caused by migration among colonies and random sampling drift.

Let $X=[0\quad 1]^S$ be the space of systems of gene frequencies, which is equipped with the product topology. Let $C(X)$ be the Banach space of all continuous functions equipped with the supremum norm and $C_0^2(X)$ be the set of all C^2-functions depending only on finite number of coordinates of X.

Let us consider the following infinite dimensional differential operator A,

$$Af(x)=\sum_{i\in S}\frac{1}{4N}x_i(1-x_i)\frac{\partial^2 f}{\partial x_i^2}+\sum_{i\in S}\Big(\sum_{j\in S}q_{ij}x_j\Big)\frac{\partial f}{\partial x_i}, \tag{1.1}$$

where $N>0$ and q_{ij} $(i, j\in S)$ are constants such that $q_{ij}\geqq 0$ for $i\neq j$ and $\sum_{j\in S}q_{ij}=0$ for each $i\in S$.

Let $\{T_t\}$ be a strongly continuous semi-group on $C(X)$ such that

$$T_t1=1 \quad\text{and}\quad T_tf\geqq 0 \qquad \text{for every } f\in C(X) \text{ satisfying } f\geqq 0, \tag{1.2}$$

and

$$T_tf-f=\int_0^t T_sAf\,ds \qquad \text{for every } f\in C_0^2(X). \tag{1.3}$$

Such a semi-group $\{T_t\}$ is uniquely determined under the following assumption,

$$\sup_{i\in S}|q_{ii}|<+\infty. \qquad \text{(cf. [10], [11]).} \tag{1.4}$$

Here N means the effective population size of each colony and q_{ij} $(i\neq j)$ means the migration rate from $j\in S$ to $i\in S$.

Then $\{T_t\}$ defines a diffusion process on X, which we call *a continuous time stepping stone model without mutation and selection.*

Discrete time stepping stone models were first proposed by M. Kimura and