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Reproducing differentials and certain theta functions on open Riemann surfaces

By

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Introduction

The purpose of the present note is to introduce the theta function on open Riemann surfaces using some fundamental Abelian differentials with finite norm, especially reproducing kernels for analytic differentials (cf. Ahlfors-Sario [1]) and square integrable normal differentials (cf. Kusunoki [4]). In § 1 we shall construct a factor of automorphy for the Fuchsian group acting on the universal covering surface of an open Riemann surface W using the reproducing kernel for $\Gamma_a(W)$, the space of all square integrable analytic differentials on W. If we replace $\Gamma_a(W)$ by $\Gamma_{ase}(W)$, then we have a factor of automorphy associated with $\Gamma_{ase}(W)$, where $\Gamma_{ase}(W)$ is the subspace of $\Gamma_a(W)$ consisting of all semi-exact elements. In § 2 we shall define the theta function induced by a factor of automorphy above and some associated functions on W which correspond to the alternating Riemann form and the Hermitian Riemann form for theta functions of finitely many variables. We shall show in § 3 that an analogue of the Riemann's theta function can be obtained from a theta function defined in § 2 by multiplying an exponential of a certain analytic double integral provided that a surface W belongs to a restricted class of Riemann surfaces. In § 4 we shall give a condition that a theta function on a parabolic open Riemann surface of positive finite genus should be continued analytically to the compact prolongation of the surface. Finally we shall show also that our theta function defined in § 2 on a plane domain bounded by a finite number of analytic Jordan curves is obtained from the Riemann's theta series associated with the double of that domain.

§ 1. Factors of automorphy associated with reproducing kernels

Let W be an open Riemann surface, ζ be a local parameter of a given point of W and z be a local parameter of a general point of W. The following are known (cf. [1], V. 18, 19, 20):