

# On the conditions for the hyperbolicity of systems with double characteristic roots, I.<sup>1)</sup>

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(Received Sept. 18, 1979)

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1) This work was supported by the Sakkokai Foundation.