§1. Introduction

We consider the Cauchy problem for hyperbolic systems with multiple characteristics of constant multiplicity. Let \( \Omega \) be a band \([0, T] \times \mathbb{R}^n \) in \( \mathbb{R}^{n+1} \). We consider the following equations in \( \Omega \),

\[
\sum_{q=1}^{N} a_{q}(x, D)u^{q}(x) = f^{p}(x), \quad p = 1, \ldots, N,
\]

where \( x = (x_0, x_1, \ldots, x_n) = (x_0, x') \in \Omega \) and \( a_{q}(x, D) \) differential operators of order \( m^{q}_{\xi} \) of which coefficients are in the Gevrey class \( \gamma^{s}(\Omega)(s \geq 1) \).

We use the notation as follows,

\[
D = (D_0, \ldots, D_n), \quad D_{k} = -\sqrt{-1} \frac{\partial}{\partial x_k},
\]

\[
\alpha = (\alpha_0, \ldots, \alpha_n), \quad \alpha_{k} \text{ integers},
\]

\[
D^{\alpha} = D_{\alpha_0}^{\alpha_0}D_{\alpha_1}^{\alpha_1} \cdots D_{\alpha_n}^{\alpha_n}, \quad |\alpha| = \sum \alpha_{k},
\]

\[
\xi = (\xi_0, \xi_1, \ldots, \xi_n); \quad \text{dual variables of } x,
\]

and \( \gamma_s(\Omega) \) consists of all functions \( f \) such that there exists positive constants \( C \) and \( A \) satisfying for any \( \alpha \),

\[
|D^{\alpha}f(x)| \leq CA^{(|\alpha|+1)}|\alpha| !^s, \quad x \in \Omega.
\]

We correspond the polynomial \( a_{q}(x, \xi) \) in \( \xi \) to a differential operator \( a_{q}(x, D) \). We denote by \( \partial_{q}^{\xi}(x, \xi) \) the homogeneous part of degree \( m^{q}_{\xi} \) of \( a_{q}(x, \xi) \). We define the total order \( m \) of \( \{a_{q}(x, D)\} \) such that

\[
m = \max_{\pi} \sum_{p=1}^{N} m^{p}_{\xi}(p),
\]

where \( \pi \) runs over all permutations of \([1, \ldots, N]\). Then it follows from Volevich’s lemma \([16]\) that there exists a pair of integers \( (t_p, s_p), \quad p = 1, \ldots, N, \) such that

\[
m^{q}_{\xi} \leq t_q - s_p, \quad (p, q) \in [1, \ldots, N]^2,
\]

\[
m = \sum_{p=1}^{N} (t_p - s_p),
\]

where \( s_p \) is the smallest positive integer equal to or greater than \( t_p - m^{p}_{\xi} \) for \( p = 1, \ldots, N \).