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Segal-Becker theorem for K_G-theory

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§ 1. Introduction.

Let G be a finite group and $\tilde{K}_G$ be the reduced equivariant K-theory of Atiyah-Segal [7]. By the Bott periodicity theorem [2], $\tilde{K}_G$ is the 0-th term of the reduced equivariant cohomology theory $\tilde{K}_G^*$ (cf. [5]). We denote by BU_G a representation space of $\tilde{K}_G^*$, that is

$$K_G(X)=\tilde{K}_G^0(X_+)=[X_+, BU_G]_G$$

for any compact G-space X, where $[\ ,\ ]_G$ denotes the set of G-homotopy classes of based G-maps. By ω we denote the complex regular representation of G. Let $\boldsymbol{CP}_G^\infty$ be the equivariant infinite dimensional complex projective space, which consists of lines in ω^∞ with a G-action induced from that of ω^∞. Then $\boldsymbol{CP}_G^\infty$ is a classifying space of G-line bundles.

The infinite loop space structure of BU_G defines an infinite loop map

$$\xi: Q_G(BU_G) \longrightarrow BU_G$$

where $Q_G(X)=\operatorname*{Colim}_n \Omega^{n\omega}\Sigma^{n\omega}X$ for any pointed G-space X. The canonical G-line bundle over $\boldsymbol{CP}_G^\infty$ defines a based map

$$j: \boldsymbol{CP}_{G+}^\infty \longrightarrow BU_G .$$

We put

$$\lambda=\xi\circ Q_G(j): Q_G(\boldsymbol{CP}_{G+}^\infty) \longrightarrow Q_G(BU_G) .$$

The infinite loop map λ defines a transformation of cohomology theories

$$\lambda_*: P_G^* \longrightarrow K_G^* ,$$

where P_G^* is an equivariant cohomology theory defined by $Q_G(\boldsymbol{CP}_{G+}^\infty)$. Then we have

Theorem 1. *For any compact G-space X,*

$$\lambda_*: P_G(X) \longrightarrow K_G(X)$$

is a split epimorphism, where $P_G(X)=\tilde{P}_G^0(X_+)$.