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Singular perturbation approach to traveling waves in competing and diffusing species models

By

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1. Introduction.

In the field of population dynamics, since Fisher's model had been presented, there have been extensive studies of reaction-diffusion equations of the form

$$\frac{\partial \bar{u}}{\partial t}=D\Delta \bar{u}+\bar{f}(\bar{u}), \tag{1.1}$$

where $\bar{u}$ and $\bar{f}$ are n dimensional vectors and D is an $n\times n$ constant matrix. It is widely known that (1.1) exhibits a variety of interesting phenomena, in spite of its simplicity. One of them is the appearance of traveling wave fronts. This type of solution is represented by the form

$$\bar{U}(z)=\bar{u}(x-ct),$$

where c is a velocity vector. This function $\bar{U}$ necessarily satisfies the following system of ordinary differential equations

$$D\bar{U}''+c\bar{U}'+f(\bar{U})=0, \tag{1.2}$$

subject to appropriate boundary conditions imposed at $z=\pm\infty$, where $'=d/dz$. When $n=1$, the existence of $\bar{U}(z, c)$ and its stability were almost completely discussed by many authors. For $n=2\sim4$, there are some results on biological models such as Nagumo's equation, Hodgikin-Huxley's equation, and Field-Noyes's equation (see, for instance, [1, 5, 12]). However, there has not been as yet any powerful general theory for any n, except topological methods developed by Conley [3].

In the framework of (1.1), we discuss here a model of two competing and diffusing species described by

$$\begin{aligned}\frac{\partial u}{\partial t}-d_1\frac{\partial^2 u}{\partial x^2}&=f_0(u, v)u\\ \frac{\partial v}{\partial t}-d_2\frac{\partial^2 v}{\partial x^2}&=g_0(u, v)v\end{aligned}\quad, \tag{1.3}$$

where u and v are the population densities of the two species. It is assumed from the competitive interaction that f_0 and g_0 satisfy