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Geometric invariants associated with flat projective structures

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Introduction

Let M be an n-dimensional manifold ($n \geqq 2$) with a projective structure $[\chi]$. It is well known that, corresponding to $[\chi]$, a projective normal Cartan connection ω is constructed uniquely on a certain principal bundle P and that the projective structure $[\chi]$ is flat if and only if ω is a flat Cartan connection [8], [10], [11]. In the present paper we shall construct, using projective Cartan connections on P, geometric invariants associated with flat projective structures on M.

In recent years the theory of secondary characteristic classes has been studied extensively and many geometric invariants have constructed for several types of geometry. Our theory is based on the method of F. W. Kamber and P. Tondeur's construction of characteristic homomorphisms for flat G-bundles with an H-reduction [7; Chap. 3]. Although a Cartan connection is not a connection in the usual sense, we can apply their method to the projective case and we construct a characteristic homomorphism ω: $H(\mathfrak{g}, K) \rightarrow H(M, \boldsymbol{R})$ ($\mathfrak{g}=\mathfrak{sl}(n+1, \boldsymbol{R})$ and $K=O(n)$) for a flat projective structure on M.

In Tanaka [11] a projective normal Cartan connection ω is constructed in the following way. First we fix an affine connection χ on a frame bundle $\tilde{P}$ which belongs to the original projective structure. Next we extend the structure group of $\tilde{P}$ to the isotropy subgroup G' of the projective transformation group of $P^n(\boldsymbol{R})$. We denote by P the extended principal G'-bundle. Then there exists uniquely a projective normal Cartan connection ω on P satisfying certain conditions (see [1]). Reversing this procedure, i.e., reducing the structure group of P to the maximal compact subgroup K of $GL(n, \boldsymbol{R})$, we can construct a DG-algebra homomorphism $(\wedge \mathfrak{g}^*)_K \rightarrow A(M)$ in the flat case and the induced cohomology map $H(\mathfrak{g}, K) \rightarrow H(M, \boldsymbol{R})$ does not depend on the choice of K-subbundles. (For the definitions, see §2).

Applying the method described in [4], we can determine the relative cohomology algebra $H(\mathfrak{g}, K)$ and we know that the invariants $\omega(x_{4k+1}) \in H^{4k+1}(M, \boldsymbol{R})$ are defined for a flat projective structure on M. It is known that two flat projective structures on M are isomorphic if and only if there is a bundle isomorphism ϕ: $P \rightarrow P$ which preserves the corresponding flat Cartan connections (cf. Theorem A in [1]). But