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On pseudo-poles of Abrikosov equation

By

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1. Introduction

In this paper we investigate the ordinary differential equation

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{\nu^2}{r^2}u = u^3 - u \qquad (r>0). \tag{1}$$

Here ν is a positive parameter. This equation is called Abrikosov equation and describes vortex lines of a superconductor of type II in the theory of superconductivity. The existence and uniqueness of a global solution satisfying $0<u\leqq 1$ have been established by Y. Kametaka, [3]. Besides this global solution there are solutions with movable infinitudes, that is, solutions blowing up at $r=R$, R being an arbitrary finite positive number. We are interested in the nature of the movable infinitudes of the equation.

If $\nu=1/3$ the equation is equivalent to the second Painlevé equation

$$\frac{d^2w}{dz^2} = 2w^3 + zw$$

by the change of variables*

$$z = -\left(\frac{3}{2}r\right)^{\frac{2}{3}}, \quad w = 2^{-\frac{1}{2}}\left(\frac{3}{2}r\right)^{\frac{1}{3}} u.$$

Therefore if $\nu=1/3$ all possible infinitudes are simple poles. However if $\nu\neq 1/3$ there appear movable infinitudes which involve logarithmic terms in the expansions. In Section 2 of this paper such infinitudes are constructed. They can be called 'pseudo-poles' after E. Hille, who emphasized that such infinitudes appear in the Thomas-Fermi and Emden's equations ([2], Chapter IX).

On the other hand it is known that Abrikosov equation has a family of solutions which are asymptotically equivalent to $a_0 r^\nu$ as $r\to +0$, a_0 being an arbitrary positive constant ([3], Theorem 5). The bounded solution mentioned above is asymptotic to $a_0(\nu)r^\nu$ with a certain value $a_0=a_0(\nu)$ of the constant. What happens if we continue the solution starting with $a_0\neq a_0(\nu)$ from $r=+0$ to the right? We will devote

* due to a private communication with Y. Kametaka.