J. Math. Kyoto Univ. (JMKYAZ)
22-4 (1983) 619–642

On local solvability of some non-kowalewskian partial differential operators

By

Takashi ŌKAJI

(Received July 17, 1981)

1. Introduction

We are concerned with local solvability of the partial differential operators. The notion of local solvability in the distribution's sense was introduced by L. Hörmander. Let Ω be a domain of $\boldsymbol{R}^n$ and P be a partial differential operator with smooth coefficients in Ω.

Definition 1. We say that P is locally solvable at the point $x \in \Omega$ if and only if there exists a neighborhood U of x such that for every $f \in C_0^\infty(U)$, there exists $u \in \mathscr{D}'(U)$ which satisfies $Pu = f$ in $\mathscr{D}'(U)$.

Let I be a interval $[-T, T]$, $D_t = \frac{1}{i}\frac{\partial}{\partial t}$, and $D_x^\alpha = \frac{1}{i}\frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \cdots \partial x_n^{\alpha_n}}$, where $\alpha = (\alpha_1, \dots, \alpha_n) \in N^n$, and $N = (0, 1, 2, \dots)$. In this paper we shall consider the local solvability of the operator

$$(1) \qquad L = D_t + P(x, t, D_x) \qquad (x, t) \in \Omega \times I$$

, where $P(x, t, D_x) = \sum_{|\alpha| \leq m} a_\alpha(x, t) D_x^\alpha$, and $a_\alpha(x, t) \in C^\infty(\Omega \times I)$. When $m = 1$, local solvability of L is almost completely decided. (L. Nirenberg and F. Treves [17]). So we consider the case $m \geqslant 2$. In this case, L becomes non-kowalewskian operator. In non-degenerate case, hypoellipticity of parabolic system has been proved by S. Mizohata. In degenerate case, hypoellipticity and well-posedness for Cauchy problem is considered by many people. Some of their works give us some information for L to be locally solvable. But we have little knowledge of necessary condition for L to be locally solvable. For example, Y. Kannai has showed that

$$L_1 = D_t + itD_x^2$$

is hypoelliptic but not locally solvable at the origin, and R. Rubinstein has showed that

$$L_2 = D_t - it^n D_x^2 + it^m D_x \qquad (n;\ \text{even})$$