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A remark on the Hölder continuity of the solution for a certain elliptic equation with irregular coefficients

By

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§0. A solution for an elliptic equation, even it may satisfy the equation in a weak sense, is expected to have certain regularity properties in the interior of the domain in question. In this paper we treat the elliptic equation of the following form

$$-\triangle u+\sum_{j=1}^{n} b_j(x)\frac{\partial u}{\partial x_j}+c(x)u=f(x)\,, \tag{0.1}$$

where $\triangle$ denotes the Laplace operator in R^n. We denote by $B(x, r)$ the ball in R^n with the center x and radius r, and $B_R=B(0, R)$ in abbreviation. Since we are interested in the interior regularity properties of the solution u, we may confine our considerations within a neighborhood B_{R_0} of the origin. We impose the following conditions on b_j, c and f.

c and f belong to $L^1(B_{R_0})$, b_j's belong to $L^2(B_{R_0})$ and there exist constants B, C, F and θ $(0<\theta\leqslant 1)$ such that the following inequalities hold

$$\left\{\begin{array}{l} \sum_{j=1}^{n}\int_{B(x,r)\cap B_{R_0}}|b_j(y)|^2\,dy\leqslant B^2r^{n-2+\theta}, \quad \int_{B(x,r)\cap B_{R_0}}|c(y)|\,dy\leqslant Cr^{n-2+\theta} \\ \int_{B(x,r)\cap B_{R_0}}|f(y)|dy\leqslant Fr^{n-2+\theta} \qquad \text{for every ball } B(x, r) \text{ in } R^n. \end{array}\right. \tag{0.2}$$

We say that $u\in H^1(B_{R_0})$ (the L^2-Sobolev space of order 1) is a weak solution of (0, 1) in B_{R_0}, when u satisfies

$$\int_{B_{R_0}}\nabla u\cdot\nabla\varphi+\sum_{j=1}^{n} b_j\frac{\partial u}{\partial x_j}\varphi+cu\varphi dx=\int_{B_{R_0}}f\varphi dx$$

for all $\varphi\in C_0^1(B_{R_0})$. Since $b_j(\partial u/\partial x_j)$ and cu^2 are integrable under the conditions (see Lemma 1.2), the above definition makes sense.

Now we state our main result in this paper.

Theorem. *Under the condition* (0.2), *if a function* $u\in H^1(B_{R_0})$ *is a weak solution of* (0.1), *then* u *is equivalent to a Hölder continuous function with*