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Classification of the double projections of Veronese varieties

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1. Introduction

Let r, s be arbitrary positive integers. The Veronese variety $V_{r,s}$ over a field k is the projective variety in $\boldsymbol{P}_k^{N-1}$, $N=\binom{r+s-1}{r-1}$, whose homogeneous coordinate ring is generated by the N monomials of degree s in r indeterminates. By a projection of $V_{r,s}$ we understand a projective variety in $\boldsymbol{P}_k^{N-d}$, $d>1$, whose homogeneous coordinate ring is generated by $N-d+1$ such monomials. In [5] Gröbner showed that the defining prime ideal of $V_{r,s}$ is perfect, i.e., the homogeneous coordinate ring of $V_{r,s}$ is Cohen-Macaulay, but certain projections of $V_{r,s}$ in $\boldsymbol{P}_k^{N-2}$ have imperfect defining prime ideals. From this phenomenon he then posed the problem of classifying projections of Veronese varieties.

There were first some efforts of Renschuch to solve this problem in [9], [10]. But the first important result is due to Schenzel, who showed in [12] which projections of $V_{r,s}$ in $\boldsymbol{P}_k^{N-2}$ have Cohen-Macaulay or Buchsbaum local rings at the vertex of their affine cones. Note that the homogeneous coordinate ring of a projective variety is Cohen-Macaulay if and only if the local ring at the vertex of its affine cone is Cohen-Macaulay (see e.g. [8, Proposition 4.10]), and that a local ring A is called Buchsbaum if for all ideals q generated by a system of parameters of A the difference $l(A/q)-e(q;A)$ between length and multiplicity is an invariant $i(A)$ of A; hence A is Cohen-Macaulay if and only if A is Buchsbaum with $i(A)=0$ (see [13], [14] for further informations). However, Schenzel's method doesn't work for the classification of the projections of $V_{r,s}$ in $\boldsymbol{P}_k^{N-d}$ $d>2$ [12, p. 396].

In this paper we shall give a complete classification of the projections of $V_{r,s}$ in $\boldsymbol{P}_k^{N-3}$ (double projections) under the same aspect by using some results on the Cohen-Macaulay or Buchsbaum property of affine semigroup rings.

Let $t_1,\dots,t_r$ be r indeterminates over k. If H is an additive semigroup in $\boldsymbol{N}^r$, one can define the semigroup ring $k[H]$ of H over k to be the subring of $k[t_1,\dots,t_r]$ generated by all monomials $t_1^{\alpha_1}\cdots t_r^{\alpha_r}$ with $(\alpha_1,\dots,\alpha_r)\in H$. Thus, if H is a finitely generated additive semigroup with zero in $\boldsymbol{N}^r$, $k[H]$ is an affine ring, hence we call H an affine semigroup. Notice that $k[H\backslash\{0\}]$ may be considered as an ideal of