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Comparison of measures on the maximal ideal space of $H^{\infty}(W)$ with applications to the Dirichlet problem

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0. Introduction

Let W be any subdomain of $C^n (n \geqq 1)$, the n-dimensional complex Euclidean space. By R^{2n} we denote the underlying real 2n-dimensional Euclidean space of C^n, as usual. Throughout this paper we shall assume that W admits either nonconstant bounded analytic functions, or nonconstant bounded harmonic functions. $H^{\infty}(W)$ will denote the commutative Banach algebra of all bounded analytic functions on W endowed with the uniform norm. $HB_R(W)$ is the order complete Banach lattice of all real-valued bounded harmonic functions in $W \subset R^{2n}$ with the sup norm topology. $M_{H^{\infty}(W)}$ will stand for the maximal ideal space of $H^{\infty}(W)$.

This paper deals with the Dirichlet problem on the Shilov boundary S of $M_{H^{\infty}(W)}$. Namely, under the appropriate conditions we investigate a positive linear map from $C_R(S)$ into $HB_R(W)$ which acts on Re $H^{\infty}(W) (\subset C_R(S))$ as the identity map. Since $HB_R(W)$ is an order complete Banach lattice, such a map as above always exists: we can apply techniques of the positive extension in Hahn-Banach's extension theorem to this problem. The solution obtained by this method, however, gives us few information on the maximality in the following sense. Let L be any solution of the problem, and consider the functional $C_R(S) \ni g \rightarrow L(g)(p)$, where p is an arbitrary, but fixed, point of W. Clearly this functional is represented by the probability measure which is uniquely determined, and is supported on S. Denote this measure by $\mathrm{d}v$. Then by the positiveness, and from Harnack's inequality we have the nonnegative kernel $Q(z, \ )$ of $L^{\infty}(\mathrm{d}v)$ such that $L(g)(z) = \int gQ(z, \ )\mathrm{d}v$ for all $g \in C_R(S)$ and for all $z \in W$. By the maximality for solutions we mean that the induced measure $\mathrm{d}v$ for p as above is a *boundary measure* in the sense of Alfsen [1]. Thus our aim in this paper is to investigate the maximal solutions of the Dirichlet problem on S in terms of Choquet order relation.

In case that W is a subdomain of the complex plane, or more generally, a Rieman surface, the author characterized a representing measure $\mathrm{d}v$ for any point of W such that $\mathrm{d}v$ is a boundary measure and has a positive kernel $Q(z, \ )$ of $L^{\infty}(\mathrm{d}v)$ with a parameter $z \in W$: $Q(z, \ )\mathrm{d}v$ satisfy the following [3].

1) For every $z \in W$, $Q(z, \ )\mathrm{d}v$ is a representing measure for z with respect to