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The spectra of 1-forms on simply connected compact irreducible Riemannian symmetric spaces

By

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Introduction

Let G/K be a simply connected compact irreducible Riemannian symmetric space and let $A^p(G/K)$ be the space of complex continuous p-forms on G/K. Then it may be natural to ask: How does $A^p(G/K)$ decompose under the canonical action of G?

For several low rank G/K, such as the spheres, the complex projective spaces, the quarternion projective spaces and the complex quadrics, the answer to this question have been given (see Gallot-Meyer [5], Ikeda-Taniguchi [8], Levy-Bruhl-Laperrière [9], [10], Strese [11] and Tsukamoto [13]).

The purpose of this paper is to decompose $A^1(G/K)$ for all simply connected compact irreducible Riemannian symmetric spaces G/K. The method used in this paper is somewhat different from that used in the above papers.

Let $A^1(G)$ be the space of complex continuous 1-forms on G. We can regard $A^1(G)$ as a G-module under the action of G induced by left translations of G. Then in a natural way, $A^1(G/K)$ may be considered as a G-submodule of $A^1(G)$. Therefore to decompose $A^1(G/K)$, we have only to express $A^1(G)$ as a sum of irreducible G-submodules and find out all the factors of this decomposition that are contained in $A^1(G/K)$. Then our problem is to determine the function that assigns to each irreducible G-module the number of factors in $A^1(G/K)$ isomorphic to this G-module.

In §1, making use of the theorem of Peter-Weyl on the representative ring of G, we reduce our problem to the following problem: For each irreducible representation $\rho: G\to GL(V^\rho)$, determine the multiplicity of the eigenvalue -1 in $(V^\rho\otimes\mathfrak{g}^c)_K$ of the involutive automorphism $\tilde{\theta}$ of $(V^\rho\otimes\mathfrak{g}^c)_K$ induced by the canonical involution θ of $\mathfrak{g}^c$ associated to the symmetric pair (G, K) (see the definitions in §1). This problem can further be reduced to a problem of the complexified Lie algebra $\mathfrak{g}^c$. In §3 we define a map of $V^\rho\otimes\mathfrak{g}^c$ onto $\mathfrak{g}^c$ that sends $(V^\rho\otimes\mathfrak{g}^c)_K$ isomorphically onto a θ-invariant subspace $\mathfrak{p}$ of $\mathfrak{g}^c$. Then the problem stated above can be reduced to the problem of determination of the multiplicity of the eigenvalue -1 of θ in $\mathfrak{p}$. In order to solve this problem, we first clarify the relation between this multiplicity and the subset $B(\Lambda)$ of non-zero roots of $\mathfrak{g}^c$ determined by the highest weight Λ of ρ.