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Metrical Finsler structures and metrical Finsler connections

Dedicated to Professor Makoto Matsumoto on the occasion of his sixtieth birthday

By

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A Finsler space is sometimes adopted as a basic concept in the theoretical physics. However, the fact that the fundamental tensor field of a Finsler space is provided from a positively homogeneous function is not always desirable for physicists, as pointed out by several authors. In fact, recently Matsumoto [8] showed an unexpected result (Corollary of Theorem 2) on four-dimensional Finsler spaces, which may be a direct consequence of such an origin of the fundamental tensor field. It seems to the author that Kern's Lagrange geometry [6] is noteworthy in this aspect. As to physical viewpoint, see Ingarden's lecture [5] and Takano's lecture [11]. Further it is suggestive that Horváth and Moór [4] again developed their theory based on a generalized metric in Moór's terminology, after their Finsler-geometric treatment of the same subject [3].

In the present paper we first define a metrical structure on a differentiable manifold as a Finsler tensor field g of type (0, 2) in Matsumoto's terminology [7] and establish the existence of a set of connections $F\Gamma$ of Finsler type which are metrical with respect to g. Based on the notion of absolute energy associated to g, we define regular Finsler structures. From a regular Finsler structure a metrical Finsler connection, called canonical by us, is uniquely determined. This Finsler connection is regarded as a generalization of the Cartan connection in case of Finsler geometry.

Almost all the theorems in this paper are proved applying the methods given by the present author and Hashiguchi [9, 10]; so the proofs are omitted.

Throughout the present paper we suppose that the contents of Matsumoto's monograph [7] are known.

§1. The metrical Finsler structures and metrical Finsler connections

Let M be an n-dimensional differentiable manifold, TM its tangent bundle and π: $TM \to M$ the natural projection. If $U \subset M$ is the coordinate neighborhood of a coordinate system (x^i), then $\pi^{-1}(U) \subset TM$ is a coordinate neighborhood, too. Let (x^i, y^i) be the coordinate system of a point $y \in \pi^{-1}(U)$, $x=(x^i)=\pi(y)$.