

Notes on theta functions for open Riemann surfaces

By

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(Communicated by Prof. Y. Kusunoki, May 29, 1982)

Introduction.

In the present paper we shall study some aspects of theta functions for open Riemann surfaces using normal behavior spaces of harmonic differential introduced by Maitani [5] and Kusunoki-Maitani [4]. For a given normal behavior space on an open Riemann surface W we consider a complex Hilbert space H of Abelian differentials spanned by fundamental differentials with the boundary behavior associated with the behavior space, and we show that there is a natural "Hermitian Riemann form" on the dual space H' of H with respect to a certain subgroup of H' which is called a "lattice" of H' . By using this Hermitian Riemann form we shall define analytic factors of automorphy on H' and we obtain the corresponding factors on a universal covering surface \hat{W} of W . Thus we have theta functions on the Hilbert space H' and those on the open Riemann surface W . If the periods of fundamental differentials in H satisfy a certain growth condition presented by Tahara [8], then it turns out that for a given factor of automorphy the space of theta functions on H' represented as complex analytic Fourier series is of one dimension. The reproducing differential of H will play a fundamental role through this paper.

In §1 we shall introduce a closed subspace H of $\Gamma_{\sigma, \sigma}$ associated with a given normal behavior space and show the intersection property of the reproducing differential $k(p, q)d\bar{p}d\bar{q}$ of H , that is,

$$\operatorname{Im}\left(\int_{C_1} d\bar{p} \int_{C_2} k(p, q)d\bar{q}\right) = \pi(C_1 \times C_2)$$

for finite cycles C_1 and C_2 . This property is a standpoint of the present study. In §2 we remark that for open Riemann surfaces we can define formally, by using reproducing differentials, analogs of the Jacobi mappings. Analytic factors of automorphy on H' , the dual space of H , and corresponding factors on a universal covering surface \hat{W} of W will be given in §3. In §4 a Riemann's theta function is defined to be an analytic relatively automorphic function of a factor of automorphy given in §3. We shall show that, under a certain growth condition for periods of fundamental differentials in H , for a given factor on H' theta functions admitting complex analytic Fourier series form a one dimensional vector space. To this end we shall investigate in the