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On the poles of the scattering matrix for two strictly convex obstacles: An addendum

By

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§ 1. Introduction

The purpose of this paper is to improve the second part of Theorem 1 of the previous paper [2]. Namely, we like to give a more precise information on the existence of the poles of the scattering matrix $\mathscr{S}(z)$. The result we want to show in this paper is

Theorem 1. *Suppose that $\mathcal{O}$ satisfies the same conditions as in Theorem* 1 *of* [2]. *Then there exists at least a pole of $\mathscr{S}(z)$ in $\{z; |z-z_j| \leqq C(|j|+1)^{-1/2}\}$ for all large $|j|$.*

As remarked in [2], in order to show Theorem 1 it suffices to prove

Theorem 2. *The operator $U(\mu)$ which is defined in Theorem* 2 *of* [2] *has at least a pole in $\{\mu; |\mu-\mu_j| \leqq C(|j|+1)^{-1/2}\}$ for all large $|j|$.*

The plan of the proof of Theorem 2 is as follows. First we shall construct an asymptotic solution $u(x, t; k)$ of the problem

$$(1.1) \qquad \begin{cases} \Box u = 0 & \text{in} \quad \Omega \times \boldsymbol{R} \\ u = m(x, t; k) & \text{on} \quad \Gamma \times \boldsymbol{R} \\ \operatorname{supp} u \subset \bar{\Omega} \times \{t; t > 0\} \end{cases}$$

for an oscillatory boundary data

$$(1.2) \qquad m(x, t; k) = e^{ik(\varphi_\infty(x)-t)} g(x) m(t)$$

following the process of [2], where φ_∞ is a phase function introduced in §3 of [2], and $g(x) \in C_0^\infty(\Gamma_1)$, $m(t) \in C_0(\boldsymbol{R})$. Then the Laplace transform $\hat{u}(x, \mu; k)$ of $u(x, t; k)$ becomes an approximation of $\hat{m}(\mu+ik)U(\mu)(e^{ik\varphi_\infty(\cdot)}g(\cdot))(x)$, and we estimate $\Delta_{C_j}\hat{u}(A(l_0), \mu; k_j)$ for $A(l_0)$ a point on the segment a_1a_2, $C_j = \{\mu; |\mu-\mu_j| = \eta\}$ $(\eta > 0)$ and $k_j = -j\pi/d$, where $\Delta_C\hat{u}$ denotes the variation of arg $\hat{u}$ along the contour C.