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On the existence of transversal homoclinic points of some real analytic plane transformation

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0. Introduction

In this paper we shall consider the following real analytic plane transformation f:

$$f(x, y)=(y+\xi(x), x),$$

$$f^{-1}(x, y)=(y, x-\xi(y)),$$

where ξ is a real analytic function belonging to X which will be define in Section 5 below. X is not vacant. For example $\xi(x)=cx(1-x)(c>0)$ is in X.

The differential of f is

$$df_{(x,y)}=\begin{pmatrix} \xi'(x) & 1 \\ 1 & 0 \end{pmatrix}$$

(x_0, y_0) is a fixed point if and only if $x_0=y_0$ and $\xi(x_0)=0$. Also, it is hyperbolic if and only if $\xi'(x_0)\neq 0$.

By the parallel transformation along the line $x=y$, its form is invariant ($\xi(x)$ tends to $\xi(x-a)$).

We assume $0=(0, 0)$ to be a hyperbolic fixed point. The eigenvalue λ of df_0 satisfies following equation:

$$\lambda^2-\xi'(0)\lambda-1=0.$$

Let s be the reflection with respect to the line $x+y=1$:

$$s(x, y)=(1-y, 1-x).$$

Conjugating f by s, we get

$$sfs^{-1}=(y, x-\xi(1-y)).$$

The identity $sfs^{-1}=f^{-1}$ holds if and only if ξ satisfies the following functional equation: