J. Math. Kyoto Univ. (JMKYAZ)
23-4 (1983) 649-694

Capillary-gravity waves for an incompressible ideal fluid

By

Hideaki YOSIHARA

(Received Aug. 15, 1982)

§1. Introduction

We shall consider nonstationary waves on the free surface of an incompressible ideal fluid when the surface tension is taken into account. The surface tension creates the pressure difference across the surface which is proportional to the mean curvature of the surface (Laplace's formula, [3] Ch. VII).

Matters relevant to the capillarity (capillary phenomena) are treated in [4]. For mathematical problems on capillary surfaces, problems determining the shape of the surface under the action of the surface tension, see, for example, [5] and papers in Pacific J. Math. 88 No. 2 (1980).

We take coordinates $y=(y_1, y_2)$ so that the fluid at rest occupies the domain

$$\{y \mid -\infty<y_1<+\infty, \quad -h+b(y_1)\leqq y_2\leqq 0\},$$

where $h=\text{const}>0$ is the mean depth and b is a given function such that $-h+b(y_1)<0$ for all y_1. The gravitational field is equal to $(0, -g)$, where $g=\text{const}$ is not necessarily positive.

The irrotational motion of the fluid is governed by equations and conditions for the density $\rho=\text{const}>0$, the velocity $v=(v_1, v_2)$, the pressure p and F defining the free surface, i.e. the domain

$$\Omega(t)=\{y \mid -\infty<y_1<+\infty, \quad -h+b(y_1)\leqq y_2\leqq F(t, y_1)\}$$

which the fluid occupies at time t. ρ, v and p satisfy the equation of motion, continuity and irrotationality, i.e. for t and y such that $y\in\Omega(t)$,

$$v_t+(v\cdot\nabla)v=-\rho^{-1}\nabla p+(0, -g), \tag{1.1}$$

$$\frac{\partial v_1}{\partial y_1}+\frac{\partial v_2}{\partial y_2}=0, \quad \frac{\partial v_2}{\partial y_1}-\frac{\partial v_1}{\partial y_2}=0, \tag{1.2}$$

where $v_t=\dfrac{\partial v}{\partial t}$, $\nabla=\text{grad}$, $v\cdot\nabla=v_1\dfrac{\partial}{\partial y_1}+v_2\dfrac{\partial}{\partial y_2}$. The fluid cannot penetrate the bottom. This means that for y such that $y_2=-h+b(y_1)$,

$$v\cdot N=0 \tag{1.3}$$