J. Math. Kyoto Univ. (JMKYAZ)
24-1 (1984) 141-162

The spectra of 1-forms on simply connected compact irreducible Riemannian symmetric spaces II

By

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(Communicated by Prof. H. Toda, October 28, 1982)

Introduction.

In the preceding paper [5], we established a method to calculate the spectra of 1-forms on simply connected compact irreducible Riemannian symmetric spaces. By use of it, we determined the spectra of 1-forms on [AIII] $SU(p+q)/S(U(p)\times U(q))$ and [G] $G_2/SU(2)\times SU(2)$. The purpose of this paper is to show the complete lists of the spectra of 1-forms on all simply connected irreducible Riemannain symmetric spaces except: (1°) Compact simple Lie groups; (2°) [AIII], [G]; (3°) [BDI, II] $SO(p+q)/SO(p)\times SO(q)$ ($q\geqq p$, $p=1, 2$). The spectra of 1-forms on [BDI, II] ($q\geqq p$, $p=1, 2$) can be seen in Ikeda-Taniguchi [4] and Tsukamoto [9]. See also Gallot-Meyer [3], Levy-Bruhl-Laperrière [6], [7] and Strese [8]. The spectra of 1-forms on compact simple Lie groups can be obtained by Theorem 2.1 and Corollary to Theorem 1.3 in [5], however they are not treated here.

In order to explain the contents of this paper, we review some fundamental notations. Let G/K be a simply connected compact irreducible Riemannian symmetric space with G simple. We denote by $\mathcal{D}(G)$ the set of equivalence classes of irreducible representations of G and $\mathcal{D}(G, K)$ the set of equivalence classes of spherical representaitons of the symmetric pair (G, K). Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{m}$ be the canonical decomposition of the Lie algebra $\mathfrak{g}$ of G associated with G/K. We choose a Cartan subalgebra $\mathfrak{t}$ of $\mathfrak{g}$ containing a maximal abelian subspace of $\mathfrak{m}$. Let $\Pi=\{\alpha_1, \cdots, \alpha_n\}$ be the set of simple roots with respect to a suitable linear order in $\mathfrak{t}$. We denote by p the Satake involution of the set $I=\{i\,|\,\alpha_i\in\Pi,\ \alpha_i\notin\mathfrak{b}\}$, where $\mathfrak{b}=\mathfrak{t}\cap\mathfrak{k}$. Let $D(G)$ be the set of dominant integral forms on $\mathfrak{t}$ and let $D(G, K)$ be the subset of $D(G)$ consisting of all highest weights of $[\rho]\in\mathcal{D}(G, K)$. The set $D(G, K)$ is given by the additive semi-group generated by the following M_i's ($i\in I$, $p(i)\geqq i$):

$$M_i=\begin{cases} 2\Lambda_i & p(i)=i,\ (\alpha_i, \Pi\cap\mathfrak{b})=\{0\}\ ; \\ \Lambda_i & p(i)=i,\ (\alpha_i, \Pi\cap\mathfrak{b})\neq\{0\}\ ; \\ \Lambda_i+\Lambda_{p(i)} & p(i)>i\ ; \end{cases}$$

where $\{\Lambda_1, \cdots, \Lambda_n\}$ stands for the set of fundamental weights.