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On unitary representations and factor sets of covering groups of the real symplectic groups

Dedicated to Professor Hisaaki Yoshizawa on his 60th birthday

By

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Introduction.

In an attempt to get the metaplectic groups of "higher degree", Kubota presented a Weil type representation for $SL(2, \boldsymbol{C})$ in the papers [7]-[10]. A similar construction of the covering groups of $SL(2, \boldsymbol{R})$ was obtained by Yamazaki [16]. Briefly speaking, they replaced the role of the Fourier transformation in the construction of so-called Weil representation [14] by that of the Fourier-Bessel transformation. In the present paper we treat the case of the real symplectic group $Sp(m, \boldsymbol{R})$, using the Bessel functions of matrix argument defined by Herz [5]. We start from a certain family of unitary operators defined on an open dense subset of $Sp(m, \boldsymbol{R})$. Then this family determines a projective unitary representation of $Sp(m, \boldsymbol{R})$. For a closer investigation of matters, we introduce a factor set for the universal covering group of $Sp(m, \boldsymbol{R})$, which can be computed explicitly. The purpose of the present paper is to study such a family of unitary operators in connection with the factor set.

Let us explain our results in more detail. Let $S_m(\boldsymbol{R})$ be the space of all $m\times m$ real symmetric matrices and P_m the space of all $m\times m$ positive definite real symmetric matrices. For $\delta>-1$, we denote by $L_\delta^2(P_m)$ the Hilbert space of square integrable functions on P_m with respect to the measure $(\det x)^\delta dx$, where dx is the restriction of usual Lebesgue measure on $S_m(\boldsymbol{R})$. We denote three types of elements in $Sp(m, \boldsymbol{R})$ by $d(a)=\begin{pmatrix} a & 0 \\ 0 & {}^t a^{-1}\end{pmatrix}$, $t(b)=\begin{pmatrix} 1 & b \\ 0 & 1\end{pmatrix}$, $d'(c)=\begin{pmatrix} 0 & -{}^t c^{-1} \\ c & 0\end{pmatrix}$ for a, $c\in GL(m, \boldsymbol{R})$ and $b\in S_m(\boldsymbol{R})$. Corresponding to these elements, we define three types of unitary operators on $L_\delta^2(P_m)$ as follows. For $\varphi\in L_\delta^2(P_m)$,

$$\boldsymbol{d}_\delta(a)\varphi(x)=\varphi({}^t a x a)|\det a|^{\delta+p} \qquad (a\in GL(m, \boldsymbol{R})),$$

$$\boldsymbol{t}_\delta(b)\varphi(x)=\varphi(x)\operatorname{etr}(\sqrt{-1}\,bx) \qquad (b\in S_m(\boldsymbol{R})),$$

$$\boldsymbol{d}'_\delta(c)\varphi(x)=\varphi^*(c^{-1}x{}^t c^{-1})|\det c|^{-\delta-p} \qquad (c\in GL(m, \boldsymbol{R})).$$

Here $p=(m+1)/2$, $\operatorname{etr}(a)=\exp(\operatorname{tr}(a))$, and φ^* is the Hankel transform of φ defined by

$$\varphi^*(x)=\int_{P_m}\varphi(y)A_\delta(xy)(\det y)^\delta dy$$