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On the syzygy part of Koszul homology on certain ideals

By

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1. Introduction.

Let A be a Noetherian local ring, m the maximal ideal of A and M a finitely generated A-module. a will always denote an ideal in A. Let $a_1, \cdots, a_r$ be a set of generators for a. Then we denote by $K.(a\,; M)$ the Koszul complex associated to a. Furthermore, $Z.(a\,; M)$ and $B.(a\,; M)$ denote the cycle and boundary of the Koszul complex respectively. For an arbitrary positive integer n we set

$$\widetilde{H}_n(a\,; M)=Z_n(a\,; M)/[Z_n(a\,; M)\cap aK_n(a\,; M)]$$

and name this module the syzygy part of the homology $H_n(a\,; M)$.

The purpose of this paper is to study some properties of the syzygy part.

Obviously there exists a canonical homomorphism of A-modules

$$H_n(a\,; M) \longrightarrow \widetilde{H}_n(a\,; M) \longrightarrow 0\,.$$

If the canonical map is injective for some integer n, then we call that $a_1, \cdots, a_r$ is $\widetilde{H}_n$-faithful (cf. [5]). A sequence of elements $a_1, \cdots, a_r$ is called a d-sequence for M if

$$(a_1, \cdots, a_{i-1})M: a_i a_j=(a_1, \cdots, a_{i-1})M: a_j$$

for every $1\leqq i\leqq j\leqq r$ and an unconditioned d-sequence for M if any permutation of $a_1, \cdots, a_r$ is a d-sequence for M (C. Huneke has defined a d-sequence for $M=A$ in [2]).

A. Simis and W. V. Vasconcelos [6] has defined $\delta(a)=[Z_1(a)\cap aA^r]/B_1(a)$ for arbitrary ideal a generated by r elements and shown that $\delta(a)=0$ if and only if the canonical homomorphism $\mathrm{Symm}(a)\rightarrow R(a)$ from the symmetric algebra to the Rees algebra is the isomorphism in degree two part of both algebras.

On the other hand, C. Huneke has discussed in [2] that if $a_1, \cdots, a_r$ is an unconditioned d-sequence for A, then $\mathrm{Symm}((a_1, \cdots, a_r))\cong R((a_1, \cdots, a_r))$ (see also [3]). Thus we can immediately see that if $a_1, \cdots, a_r$ is an unconditioned d-sequence for A, then it is $\widetilde{H}_1$-faithful.

Our first result is

Theorem 1.1. *Let $a_1, \cdots, a_r$ be an unconditioned d-sevuence for M, then*