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Existence of dualizing complexes

By

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Duality on Gorenstein rings and canonical modules of Cohen Macaulay rings are generalized if we consider a complex instead of rings or modules, and such a complex, called dualizing complex, is introduced by Grothendieck [7].

If a ring A is a homomorphic image of a Gorenstein ring, then A has a dualizing complex as is well known [5, Chapter V § 10], but other good sufficient condition of existence of dualizing complex is not known.

On the other hand Sharp showed in [21, (3.8) Theorem] that if a ring A has a dualizing complex, then A is an acceptable ring; that is (1) universally catenary, (2) formal fibers are Gorenstein and (3) for any finitely generated A-algebra B, the Gorenstein locus of Spec B is open.

Again, it follows that if A has a dualizing complex, then A has a canonical module as the initial non-zero homology module of the complex.

The purpose of this note is to investigate how extent the converse holds. We show the following;

If (S_2) holds, then acceptable rings with canonical modules have dualizing complexes (Theorem 5.2, Remark 5.3). Here both of the acceptability and the existence of canonical modules are important. Really, there exists an acceptable ring with no canonical modules (§ 6, Example 1) and also exists a non-acceptable ring with canonical modules (§ 6, Example 2).

If (S_2) does not hold but the ring is local, then slightly stronger condition on existence of canonical modules is necessary for us (Theorem 5.5).

All rings are assume to be commutative ring with identity and, except section 3, noetherian. The terminologies and notations of [5], [13] and [16] are used freely.

§ 1. Fundamental dualizing complexes.

Let A be a noetherian ring. A complex $I^\cdot$ of A-modules is called a fundamental dualizing complex [cf. 22] if

(i) I^i $(i \in \boldsymbol{Z})$ are injective A-modules

(ii) $I^\cdot$ is a bounded complex

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