J. Math. Kyoto Univ. (JMKYAZ)
24-2 (1984) 381-385

A note on the Segal-Becker type splittings

Dedicated to Professor Minoru NAKAOKA on his sixtieth birthday

By

Akira KONO

(Received January 19, 1983)

§1. Introduction

For a pointed space X, we define an infinite loop space $Q(X)$ by $Q(X)=\operatorname{Colim}_n \Omega^n \Sigma^n X$. If X is an infinite loop space, then there is an infinite loop map $\xi: Q(X)\to X$ called the structure map.

The natural inclusion $j: BU(1)=\boldsymbol{CP}^\infty\to BU$ and the structure map $\xi: Q(BU)\to BU$ of BU defined by the Bott periodicity theorem define an infinite loop map

$$\lambda: Q(\boldsymbol{CP}^\infty) \longrightarrow BU.$$

Quite similarly we can define $\lambda: Q(\boldsymbol{HP}^\infty)\to BSp$ and $Q(BO(2))\to BO$. In (7) Segal showed that λ has a splitting, that is there is a map $\varepsilon: BU\to Q(\boldsymbol{CP}^\infty)$ such that $\lambda\circ\varepsilon$ is a homotopy equivalence. On the other hand in (2) Becker constructed a splitting explicitly.

In this paper we give another construction of the splitting $\varepsilon_{\boldsymbol{C}}$ using the representation theory of compact Lie groups.

For the real and quaternionic cases, we can construct the splittings $\varepsilon_{\boldsymbol{R}}: BO\to Q(BO(2))$ and $\varepsilon_{\boldsymbol{H}}: BSp\to Q(HP^\infty)$ quite similarly.

The natural maps $BU\to BSp$ and $\boldsymbol{CP}^\infty\to \boldsymbol{HP}^\infty$ defined by the natural inclusion $\boldsymbol{C}\hookrightarrow\boldsymbol{H}$ are denoted by j' and the natural maps $BU\to BO$ and $BU(1)\to BO(2)$ defined by $\boldsymbol{C}\cong\boldsymbol{R}^2$ are denoted by r. Then the purpose of this paper is to show

Theorem. *The diagrams*

$$\begin{array}{ccc} BU & \xrightarrow{j'} & BSp \\ {\scriptstyle\varepsilon_{\boldsymbol{C}}}\downarrow & & \downarrow{\scriptstyle\varepsilon_{\boldsymbol{H}}} \\ Q(\boldsymbol{CP}^\infty) & \xrightarrow[Q(j')]{} & Q(\boldsymbol{HP}^\infty) \end{array}$$

$$\begin{array}{ccc} BU & \xrightarrow{r} & BO \\ {\scriptstyle\varepsilon_{\boldsymbol{C}}}\downarrow & & \downarrow{\scriptstyle\varepsilon_{\boldsymbol{R}}} \\ Q(\boldsymbol{CP}^\infty) & \xrightarrow[Q(r)]{} & Q(BO(2)) \end{array}$$