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On convergence of holomorphic abelian differentials on the Teichmüller spaces of arbitrary Riemann surfaces

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Introduction

On the Teichmüller space of an arbitrary Riemann surface, we can consider at least two kinds of convergence of holomorphic abelian differentials. The one of them is concerned with the distortion estimates with respect to the Dirichlet norm, and is, in a sense, the most natural kind of convergence, which we call the metrical convergence (, cf. Definition 2). The other is concerned with the geometrical structure of the squares of abelian differentials, which we call the geometrical convergence (, cf. Definition 3).

We have already investigated the relation between these two kinds of convergence in the case of compact surfaces ([11]). In this paper, we will treat with the case of general surfaces and show that the geometrical convergence implies the metrical one in the case of square integrable differentials (Theorem 2). We also give sufficient conditions under which the metrical convergence implies the geometrical one (Theorems 3 and 4).

§1 is preliminaries from the theory of Teichmüller spaces and quasi-conformal mappings. The definitions of two kinds of convergence and main theorems are stated and proved in §2. Finally, as applications of main theorems, we will show in §3 that several fundamental differentials converge both metrically and geometrically.

§1. Preliminaries on the theory of Teichmüller spaces

Let a Riemann surface R^* be arbitrarily given. Then consider all pairs (R, f) of a Riemann surface R and a quasiconformal mapping f from R^* onto R. We say that (R_1, f_1) and (R_2, f_2) are equivalent if $f_2 \circ f_1^{-1}$ is homotopic to a conformal mapping from R_1 onto R_2. *The Teichmüller space* $T(R^*)$ is, by definition, the set of all equivalence classes of pairs as above. The Teichmüller space $T(R^*)$ has the usual Teichmuller metric, cf. [1] Ch. VI, and we call the topology induced by this metric