J. Math. Kyoto Univ. (JMKYAZ)
24-2 (1984) 285-303

On an operator U_χ acting on the space of Hilbert cusp forms

By

Hiroshi SAITO

(Received DEC. 17, 1982)

§0. Introduction

In a previous paper [8], we introduced an operator U_χ acting on the space of cusp forms of one variable for a Dirichlet character χ satisfying a condition, and showed that U_χ's satify $U_\chi U_{\chi'}=U_{\chi\chi'}$. By means of U_χ, we defined a decomposition of the space of cusp forms into subspaces stable under Hecke operators, and gave trace formulas of Hecke operators on each subspace. The purpose of this paper is to generalize this result to the case of Hilbert cusp forms over a totally real algebraic number field F. In [9], we have given such a formula in a special case without proof, and discussed a numerical example in the case where $F=\boldsymbol{Q}(\sqrt{5})$. A trace formula in a general case will be given in §2.

Notation. Let $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, and $\boldsymbol{C}$ denote the ring of national integers, the field of rational numbers, the field of real numbers, and the field of complex numbers. Let $\boldsymbol{H}$ denote the Hamilton quaternion algebra over $\boldsymbol{R}$. For an associative algebra R, let $M_r(R)$ denote the ring of r by r matrices with coefficients in R. For an associative algebra R with a unit, we denote by $R^\times$ the group of invertible elements.

§1. Operator U_χ

Let F be a totally real algebraic number field of degree g, and $\mathfrak{o}$ the ring of integers of F. For a place v of F, let F_v denote the completion of F at v and for a finite place $v=\mathfrak{p}$, let $\mathfrak{o}_\mathfrak{p}$ denote the ring of integers in $F_\mathfrak{p}$. Let F_A denote the adele ring of F and F_∞ (resp. F_f) the infinite part (resp. the finite part) of F_A. Then $F_\infty\simeq\boldsymbol{R}^g$. Let D be a quaternion algebra over F with the discriminant $\mathfrak{d}$. For infinite places $v_1,\dots, v_g$ of F, we assume D is unramified at $v_1,\dots, v_r$ and ramified at $v_{r+1},\dots, v_g$. The multiplicative group $D^\times$ can be seen the $\boldsymbol{Q}$ rational points of an algebraic group G over $\boldsymbol{Q}$. Let G_A denote the adelization of G and G_∞ (resp. G_f) the infinite part (resp. the finite part) of G_A. Then, there is an isomorphism

$$G_\infty\simeq GL_2(\boldsymbol{R})^r\times \boldsymbol{H}^{\times g-r}.$$