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A note on the local solvability of the Cauchy problem

Dedicated to Professor Sigeru MIZOHATA on his sixtieth birthday

By

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1. Introduction and results

In this note, we improve some results in the previous paper [3]. Let $p(x, D)$ be a differential operator of order m with coefficients in $\gamma^{(s)}(V)$, where V is a neighborhood of the origin in $\boldsymbol{R}^{n+1}$,

$$x=(x_0, x_1,\dots, x_n), \quad D=(D_0, D_1,\dots, D_n), \quad D_j=\frac{1}{i}\frac{\partial}{\partial x_j},$$

and $\gamma^{(s)}(V)$ denotes the set of all functions $f(x)\in C^\infty(V)$ such that for any compact set K in V, there are constants C, A with

$$|D^\alpha f(x)|\leqq CA^{|\alpha|}(|\alpha|!)^s, \ x\in K,$$

for all multi-indexes $\alpha\in N^{n+1}$.

By the definition, $\gamma^{(1)}(V)$ coincides with the set of real analytic functions in V. For convenience sake, we set $\gamma^{(\infty)}(V)=C^\infty(V)$. We denote by $p_m(x, \xi)$ the principal symbol of $p(x, D)$, and suppose that the hyperplan $\{x_0=0\}$ is non-characteristic for $p(x, D)$. Heareafter it will be assumed that $p_m(x, 1, 0,\dots, 0)=1$. Let us consider the following problem.

$$(p, \phi(x'))_\mu;\begin{cases} p(x, D)u=0 \\ D_0^j u(0, x')=0, \ 0\leqq j\leqq \mu-1, \\ D_0^\mu u(0, x')=\phi(x'), \end{cases}$$

where $x'=(x_1,\dots, x_n)$, $0\leqq\mu\leqq m-1$. Then we have

Theorem 1.1. *Let $s=\infty$. Suppose that the characteristic equation $p_m(0, \xi_0, \hat{\xi}')=0$, $\hat{\xi}'=(1, 0,\dots, 0)$ has μ real and ν non-real roots ($\mu+\nu=m$, $\nu\geqq 1$). Then there is a sequence of positive number $\{C_n\}$ with the following property: let $g(x_1)$ be any C^0-function defined near the origin for which $(p, g(x_1))_\mu$ has a local C^m-solution near the origin. Then $g(x_1)$ is C^∞ in a neighborhood of the origin and we have*