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On the cohomology mod 2 of E_8

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§ 1. Introduction

Let E_8 be the compact, connected, simply connected, simple Lie group of type E_8. As is well known that E_8 is a closed 248 dimensional manifold which is rational homotopy equivalent to

$$S^3 \times S^{15} \times S^{23} \times S^{27} \times S^{35} \times S^{39} \times S^{47} \times S^{59}.$$

The mod 2 cohomology ring of E_8 was determined by Araki and Shikata as follows:

Theorem (Araki-Shikata [1]). *As an algebra over the* mod 2 *Steenrod algebra*

$$H^*(E_8; F_2) = F_2[x_3, x_5, x_9, x_{15}]/(x_3^{16}, x_5^8, x_9^4, x_{15}^4)$$
$$\otimes \Lambda(x_{17}, x_{23}, x_{27}, x_{29}),$$

where $\deg x_i = i$, $x_5 = Sq^2 x_3$, $x_9 = Sq^4 x_5$, $x_{17} = Sq^8 x_9$, $x_{23} = Sq^8 x_{15}$, $x_{27} = Sq^4 x_{23}$ *and* $x_{29} = Sq^2 x_{27}$.

They made elaborated calculations of the Bott Samelson K-cycles and so details of the proof is not published. The purpose of this paper is to give a simple proof of the above theorem.

First we determine $H^*(\tilde{E}_8; F_2)$ for $* \leq 31$, where $\tilde{E}_8$ is the 3-connective fibre space of E_8. Next we prove that $\dim H^*(E_8; F_2) \geq 2^{15}$. Finally using the cohomology Serre spectral sequence for the fibering $\tilde{E}_8 \overset{k}{\rightarrow} E_8 \rightarrow K(Z, 3)$, $H^*(E_8; F_2)$ is determined. To prove the above theorem, we use the following well known facts:

Theore 1.1 (Bott [5]). *If G is a compact, connected, simply connected Lie group, then $H_*(\Omega G; Z)$ is torsion free.*

Theorem 1.2 (Borel-Siebenthal [4]). *The group E_8 contains a closed, connected subgroup U of local type A_8.*

Theorem 1.3 (Cartan [7]). *The group E_8 contains a closed, connected subgroup V of local type D_8 satisfying*

(1) *the center of V is of order* 2,