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Existence and uniqueness of classical solutions for certain degenerated elliptic equations of the second order

By

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§ 0. Introduction

Let Ω be a bounded domain in $\boldsymbol{R}^n$ with smooth boundary. The operators $\mathscr{A}$ which we shall treat in this memoire are of second order, linear, elliptic in the interior of Ω and degenerated only in normal direction at each point of the boundary. Under some assumptions on $\mathscr{A}$, the existence and uniqueness of the classical solution u of the equation $\mathscr{A}u=f$ will be shown for any given function f with certain Hölder continuity up to the boundary. We impose no boundary condition because we assume the "entrance property" of the boundary with respect to $\mathscr{A}$.

There are many authors who have studied various types of degenerated elliptic equations. Baouendi [2] treated the equations degenerated at the boundary, but for which the boundary is non-characteristic. Baouendi-Goulaouic [3] studied the equations degenerated in all directions at the boundary. The main tool in these two works is the elliptic regularization initiated by Oleinik (see Oleinik-Radkevic [7]) and the theory of interpolation in L^2 framework.

Recently, Goulaouic-Shimakura [6] studied the same class of operators as in [3] in the Hölder spaces. And Graham [10] studied the Dirichlet problems for Bergman Laplacian also in some Hölder spaces. Our interest in this memoire is to study the same type of operators as in the Chapter V of Graham's article. But the Hölder spaces with which we work are not the same because of the difference of the boundary conditions. Our method is, as in [6] and [10], to make use of the elementary solution for the simplest model of our operators.

In §1, we consider the model L_α in the half-space, and explain the non-isotropic degeneracy at the boundary. In §2, we describe the general setting of our equations in a bounded domain, and state the main results. In this work, some a priori inequalities of Schauder type for solutions are essential. In §3, we reduce these inequalities to the case of the half-space. And the a priori inequalities in the half-space are finally established in §5. The §4 is devoted to introduce the elementary solutions of L_α and $L_\alpha+\lambda$. The results on the existence and uniqueness of the