

On analytic and geometric properties of Teichmüller spaces

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Introduction

As is well known L. Bers [9] initiated the investigation of boundary groups of Teichmüller spaces. Afterwards many authors have been studying Teichmüller spaces and their boundaries ([1], [2], [16], [19], [20] etc.). Recently some geometric methods and results related to 3-manifolds are used in order to investigate Kleinian groups and their Teichmüller spaces (cf. Thurston [23]).

In contrast with the methods in these studies, we shall investigate, in this paper, the boundaries of Teichmüller spaces by using the methods familiar in the complex function theory. Namely, our main tools are the Grunsky's inequality and some theorems on bounded analytic functions in the unit disk, e.g. the Fatou's theorem and the Riesz' one. The method using the Grunsky's inequality was motivated by the recent work of Žuravlev [24].

In the first part of this paper, we shall show a geometric property of Teichmüller spaces and the holomorphic convexity with respect to a family of holomorphic functions (Corollary 1, Theorem 4).

In the second part of this paper, we shall investigate the boundary behaviour of holomorphic mappings of the unit disk to a Teichmüller space (Theorem 5), and consider the boundary approach in Teichmüller disks as the special case (Theorem 6). Further, we shall study the boundary behaviour of periods of holomorphic differentials of the first kind as functions of the Teichmüller space (Theorem 8).

§1. The Bers' embedding of Teichmüller spaces

Let G be a non-clementary Fuchsian group acting on the unit disk Δ . We denote by $Q_n(G)$ the set of all quasiconformal self-mappings of Δ that are compatible with G and leave $1, \pm i$ fixed. The Teichmüller space $T(G)$ of G is the set of all $w|_{\partial\Delta}$ with $w \in Q_n(G)$. The Teichmüller space $T(G)$ is a metric space with the Teichmüller metric $t_{T(G)}$. In particular, we call $T = T(\{1\})$ the universal Teichmüller space and denote by t_T the Teichmüller metric on T . If G is of the first kind, then $T(G)$ is identified with the set of all Fuchsian groups which are quasiconformal