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Gevrey well-posedness for a class of weakly hyperbolic equations

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§ 1. Introduction.

In this work we shall deal with the Cauchy problem

$$(1)\qquad \begin{cases} u_{tt}=\sum_{i,j}^{1,n}(a_{ij}(t,x)u_{x_j})_{x_i} & \text{on}\quad \boldsymbol{R}^n_x\times[0,\ T] \\ u(0,x)=\varphi(x) & (a_{ij}=a_{ji}) \\ u_t(0,x)=\psi(x) & \end{cases}$$

under the *weak hyperbolicity* condition

$$(2)\qquad \sum_{i,j}^{1,n} a_{ij}(t,x)\xi_i\xi_j\geqslant 0 \qquad \forall\xi\in\boldsymbol{R}^n.$$

We shall say that problem (1) is well-posed in some space $\mathscr{F}$ of functions or functionals on $\boldsymbol{R}^n$ if for any φ, ψ in $\mathscr{F}$ it admits one and only one solution u in $C^1([0,\ T],\ \mathscr{F})$.

It is known (see [2]) that the weakly hyperbolic equation $u_{tt}=a(t)u_{xx}$ may be not well-posed in C^∞, even if $a(t)\in C^\infty([0,\ T])$; therefore, we shall study problem (1) in the Gevrey classes $\gamma^{(s)}_{loc}$.

We shall prove the following

Theorem 1. *Let us consider problem (1) under the hypothesis (2). Let us suppose that the coefficients $a_{ij}(t,\ x)$ fulfill the following conditions:*

i) *There exists a $\sigma\geqslant 1$ such that, $\forall K\Subset\boldsymbol{R}^n_x$, the mapping*

$$\xi\longrightarrow\Big[\sum_{i,j}^{1,n} a_{ij}(t,x)\xi_i\xi_j\Big]^{1/\sigma}$$

is a continuous mapping from the sphere $S^n=\{\xi\in\boldsymbol{R}^n: |\xi|=1\}$ into the space $BV([0,\ T];\ L^\infty(K))$.

ii) *$\forall K\Subset\boldsymbol{R}^n_x$ there exist some positive constants Λ_K, A_K such that*

$$(3)\qquad ||D^\alpha_x a_{ij}(t,x)||_{L^\infty(K)}\leqslant\Lambda_K A_K^{|\alpha|}(|\alpha|!)^s$$