J. Math. Kyoto Univ. (JMKYAZ)
24-4 (1984) 725-740

On Ahlfors' weak finiteness theorem

By

Hiromi OHTAKE

(Received September 3, 1983)

§ 0. Introduction.

Let $\boldsymbol{R}^n$ be the n-dimensional euclidean space ($n \geqq 2$), and set $\boldsymbol{H}=\boldsymbol{H}^n = \{x \in \boldsymbol{R}^n; x_n > 0\}$, then $\boldsymbol{H}$ becomes the n-dimensional hyperbolic space with respect to the hyperbolic metric $\rho(x)|dx|$, where $\rho(x) = x_n^{-1}$. And let Γ be a group of isometries of $\boldsymbol{H}$, which acts discontinuously on $\boldsymbol{H}$. L. V. Ahlfors showed, in his lecture note [5], the weak finiteness theorem: if Γ is finitely generated, then the dimension of a certain class $Q(\Gamma)$ of mixed tensor densities, automorphic under Γ, is finite, which is an extension to higher dimensions of analytic parts of his famous finiteness theorem [1].

Our main aim is to introduce another certain class $\tilde{Q}(\Gamma)$ containing $Q(\Gamma)$, for which the dimension of $\tilde{Q}(\Gamma)$ is still finite (Corollary 3). In order to investigate $\tilde{Q}(\Gamma)$, we shall study a class $P(\Gamma)$ of quasiconformal deformations and derive properties of $\tilde{Q}(\Gamma)$ from those of $P(\Gamma)$ (Theorems 5 and 6, and Corollary 4).

In §1 we shall define some notations and state Ahlfors' weak finiteness theorem. In §2 we shall study quasiconformal deformations, and derive some new facts (Theorems 3 and 4). In §3 we shall state our main results, which will be proven in §5, after providing some lemmas in §4. And in the last §6 we shall state some remarks for the case $n=3$, particularly that our class $\tilde{Q}(\Gamma)$ turns out to be 0-dimensional.

The author wishes to express his deepest gratitude to Professor Y. Kusunoki for his encouragement and valuable comments, and for bringing this problem to author's attention. And the author also thanks to Drs. M. Taniguchi and H. Shiga, and Mr. M. Masumoto for their advices and comments during the preparation of this paper.

§ 1. Notations and Ahlfors' weak finiteness theorem.

By column vectors we denote the points in $\boldsymbol{R}^n$, and by ${}^tX$ the transpose of a matrix X.