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From associated graded modules to blowing-ups of generalized Cohen-Macaulay modules

By

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1. Introduction.

Throughout this paper, A denotes a local ring with maximal ideal $\mathfrak{m}$ and M a generalized Cohen-Macaulay module over A, i.e. $d:=\dim M>1$ and $\ell(H^i_{\mathfrak{m}}(M))<\infty$ for $i=0,\dots,d-1$, where $H^i_{\mathfrak{m}}(M)$ denotes the ith local cohomology module of M with respect to $\mathfrak{m}$.

Let $I(M)$ denote the maximum of the differences

$$I(\mathfrak{q};M):=\ell(M/\mathfrak{q}M)-e(\mathfrak{q};M),$$

where $\mathfrak{q}$ runs through all parameter ideals of M and $e(\mathfrak{q};M)$ is the multiplicity of M with respect to $\mathfrak{q}$. Then M being a generalized Cohen-Macaulay module just means that $I(M)<\infty$ [2].

If $\mathfrak{q}=(a_1,\dots,a_d)$ and $I(\mathfrak{q};M)=I(M)$, we call $a_1,\dots,a_d$ a standard system of parameters of M. In [9] we have shown that standard systems of parameters enjoy many interesting properties. For instance, $a_1,\dots,a_d$ is a standard system of parameters of M if and only if by every permutation, $a_1^{n_1},\dots,a_d^{n_d}$ is a d-sequence of M for all positive integers $n_1,\dots,n_d$. The notion of d-sequences was introduced by C. Huneke and has been proved as useful in different topics of Commutative Algebra [5].

It is known that there exist ideals $\mathfrak{a}$ of A such that every system of parameters of M contained in $\mathfrak{a}$ is standard. Such ideals are called M-standard ideals. In particular, M is a Buchsbaum module if and only if $\mathfrak{m}$ is a M-standard ideal, see [6] and [7] for more informations on the theory of Buchsbaum modules.

It should be mentioned that all these notions can be extended over a noetherian graded ring with unity which has only one maximal graded ideal.

Let $\mathfrak{a}$ be an ideal of A with $\ell(M/\mathfrak{a}M)<\infty$. Then $\mathfrak{a}$ is M-standard in the following cases:

(1) $\mathfrak{a}$ is generated by a standard system of parameters of M [9, Corollary