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Mixed problem for evolution systems

By

Reiko SAKAMOTO

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In the preceding paper ([1]), we studied a general initial-boundary value problem for single evolution equations. For example, such a type of equations are derived from the system of equations in complessible fluid dynamics. But, it seems more natural to construct a theory of mixed problems for systems of evolution equations, which we shall consider in this paper, making use of the analysis in the preceding paper. The idea of the framework for systems owes to the one for elliptic systems ([2]).

§1. Problems & Assumptions

1.1. Problems. Let $L(t, x, y; D_t, D_x, D_y)$ be a $N\times N$-matrix $B(t, y; D_t, D_x, D_y)$ be a $m_+\times N$-matrix, whose entries are linear partial differential operators, that is,

$$L(t, x, y; \tau, \xi, \eta)=(l_{ij}(t, x, y; \tau, \xi, \eta))_{i,j=1,\dots,N},$$

$$B(t, y; \tau, \xi, \eta)=(b_{ij}(t, y; \tau, \xi, \eta))_{i=1,\dots,m,\ j=1,\dots,N},$$

where l_{ij}, b_{ij} are polynomials with respect to (τ, ξ, η) with $\mathcal{B}^\infty$-coefficients in $(t, x, y)\in R^1\times R^1\times R^{n-1}$, where we assume these coefficients are constant outside a ball in R^{n+1}.

Our problem is to seek a vector valued solution with length N, satisfying

$$\text{(P)}\qquad \begin{cases} L(t, x, y; D_t, D_x, D_y)u=f & \text{in } (-\infty, T)\times R_+^n, \\ B(t, y; D_t, D_x, D_y)u|_{x=0}=g & \text{on } (-\infty, T)\times R^{n-1}, \\ u=0 \quad \text{for } t<0. \end{cases}$$

where f, g are vecter valued given functions with length $\{N, m_+\}$ satisfying $f=0$, $g=0$ for $t<0$. Our main result is

Theorem. *Under the assumptions* (A.1)-(A.4), (B.1)-(B.3), (A*.1) *and* (B*.1)-(B*.2), *the problem* (P) *is H^∞-well posed.*

1.2. Pincipal part of L. Let $\{p^{(i)}\}_{i=1,\dots,l}$ be given integers satisfying

$$p^{(1)}>p^{(2)}>\cdots>p^{(l)}\geqq 1.$$