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Stochastic differential equations of jump type and Lévy processes in diffeomorphisms group

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Introduction.

Consider a stochastic differential equation with respect to Brownian motions. It is a known fact that the solution defines a stochastic flow of homeomorphisms, provided that coefficients of the equation are Lipschitz continuous. See e.g. Bismut [2], Kunita [6]. Hence the solution defines a Brownian motion in homeomorphisms group G. Recently, Brownian motion in homeomorphisms group itself is studied a lot. See Harris [4]. Le Jan [9], Baxendale [1], Kunita [7] and Le Jan-Watanabe [10]. They characterize it by the infinitesimal mean and covariance, called local characteristics.

The purpose of this paper is to study the related problem for jump processes. The stochastic differential (integral) equations considered in this paper is based on the Lévy process $X_t=X_t(x)$, $x\in R^d$ (process with independent increments, continuous in probability), which takes values in the vector space of continuous maps (or continuous vector fields). We call it a C-valued Lévy process. In Section 1, we study systematically the C-valued Lévy process. We introduce the system (a, b, ν, U) which characterizes the law of C-valued process and discuss the existence problem of C-valued Lévy process associated with a given characteristics. The Lévy process with values in the vector space of C^m-maps is also considered.

In Section 2, we consider the stochastic differential equation based on C-valued Lévy process $X_t(x)$. The equation is written in short by $d\xi_t=dX_t(\xi_{t-})$ or $d\xi_t/dt=X_t(\xi_{t-})$. Hence it is a natural extension of a stochastic differential equation defined by a finite dimensional Brownian motion and a Poisson point process. Generally, the solution does not define a stochastic flow of homeomorphisms, owing to the jump part of the C-valued Lévy process. In fact, we prove that the solution defines a *Lévy process with values in the semigroup of continuous maps*, under Lipschitz conditions to the characteristics of the C-valued Lévy process. Further, if the characteristics are smooth, then the solution defines a Lévy process with values in the semigroup of smooth maps. In order that the solution defines a Lévy process with values in homeomorphisms group (or dif-

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