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Phase transition in one-dimensional Widom-Rowlinson models with spatially inhomogeneous potentials

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In a preceding paper [3], we discussed phase transition in one-dimensional Ising models. In the present paper, we consider the one-dimensional Widom-Rowlinson models with the formal Hamiltonian:

$$H(\boldsymbol{\sigma})=\sum_{k\in Z} J(\sigma_k \sigma_{k+1})-\sum_{k\in Z} h_k \sigma_k^2,$$

where $\boldsymbol{\sigma}=(\sigma_k)_{k\in Z}\in\{-1, 0, +1\}^Z$ and

$$J(\sigma)=\begin{cases} +\infty, & \text{if } \sigma=-1, \\ 0, & \text{if } \sigma\neq -1. \end{cases}$$

As for the Widom-Rowlinson models in higher dimensions, see [4-8].

Let $q^{[n,m]}_{\sigma_{n-1},\sigma_{m+1}}$ be the conditional Gibbs distribution in the interval $[n, m]$ with the boundary conditions σ_{n-1} and σ_{m+1}. We show later that the limit $\lim_{\substack{n\to-\infty \\ m\to+\infty}} q^{[n,m]}_{\tau',\tau}$ exists for any constant boundary conditions $\sigma_{n-1}=\tau'$ and $\sigma_{m+1}=\tau$. Put

$$q_{\tau',\tau}=\lim_{\substack{n\to-\infty \\ m\to+\infty}} q^{[n,m]}_{\tau',\tau}.$$

Let $\mathcal{G}(h)$ be the set of Gibbs distributions with the potentials J and $h=(h_k)_{k\in Z}$. Let $\mathcal{G}_{ex}(h)$ be the set of extremal measures of the convex set $\mathcal{G}(h)$. It is well known that

$$\mathcal{G}_{ex}(h)\subset\{q_{\tau',\tau}\,;\,\tau',\ \tau=0,\ \pm 1\}.$$

We prove the following Theorems.

Theorem 1. *Put*

$$\mathcal{M}_{+\infty}(h)=\begin{cases} \{-1, +1\}, & \text{if } \sum^{+\infty} e^{-h_k}<+\infty, \\ \{0\}, & \text{if } \sum^{+\infty} e^{-h_k}=+\infty. \end{cases}$$

A set $\mathcal{M}_{-\infty}(h)$ is defined analogously. Put

$$\mathcal{M}(h)=\mathcal{M}_{-\infty}(h)\times\mathcal{M}_{+\infty}(h).$$

The set $\mathcal{G}_{ex}(h)$ is isomorphic to $\mathcal{M}(h)$. The mapping