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Sobolev spaces of Wiener functionals and Malliavin's calculus

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Introduction.

The notion of Sobolev spaces of Wiener functionals was first introduced by D.W. Stroock [10] and I. Shigekawa [7] to formulate Malliavin's calculus rigorously and study it systematically. However it cannot be denied that their Sobolev spaces sometimes appeared very complicated. It is mainly because they should have dealt with both the derivative operator D and its dual D^*, or the Ornstein-Uhlenbeck operator $L(=-D^*D)$, not only on L^2 but also on all L^p-spaces over the Wiener space. But in 1982, P.A. Meyer pointed out the possibility to remove those apparent complications; that is, he proved the equivalence of the two norms defined in terms of L and D respectively. ([3], [4])

In the present paper, we first aim to develop Meyer's results and prove the equivalence among several Sobolev-type norms. In doing this, there are two useful tools; the Wiener chaos decomposition of L^2 and the hypercontractivity of the Ornstein-Uhlenbeck semigroup. Combining these two, we follow Shigekawa's idea to prove Theorem 1.1, which offers a sufficient condition for a linear operator to be bounded on L^p.

Next we construct the Sobolev spaces of Wiener functionals and discuss their properties. In particular, our definition allows of negative indices and such spaces contain what we call generalized Wiener functionals. In this context, we consider the composition of Schwartz's distributions and Wiener functionals, which was first studied by S. Watanabe [11]. This presents another approach to Malliavin's calculus.

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1. Basic notions.

Let (W, H, μ) be an *abstract Wiener space.* i.e., W is a separable Banach space, H is a separable Hilbert space densely and continuously imbedded in W, and μ is a Gaussian measure on W with mean 0 satisfying the condition,

$$\int_W (l, w)(l', w)\mu(dw)=\langle l, l'\rangle_H, \qquad l, l'\in W^*\subset H^*=H$$