J. Math. Kyoto Univ. (JMKYAZ)
25-1 (1985) 21-30

On the endomorphism ring of the canonical module

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(Communicated by Prof. Nagata, Nov. 9, 1983)

Introduction.

A ring will always mean a commutative ring with unit. Let R be a noetherian ring, M a finitely generated R-module and N a submodule of M. We denote by $\mathrm{Min}_R(M)$ the set of all minimal elements in $\mathrm{Supp}_R(M)$. In the case where M is of finite dimension, we put $\mathrm{Assh}_R(M)=\{\mathfrak{p}\in\mathrm{Ass}_R(M) \mid \dim R/\mathfrak{p}=\dim M\}$ and $U_M(N)=\cap Q$ where Q runs through all the primary components of N in M such that $\dim M/Q=\dim M/N$. Let T be an R-module and $\mathfrak{a}$ an ideal of R. $E_R(T)$ denotes the injective envelope of T and $H_{\mathfrak{a}}^i(T)$ is the i-th local cohomology module of T with respect to $\mathfrak{a}$. A semi-local ring means a noetherian ring with a finite number of maximal ideals and a local ring is a semi-local ring with unique maximal ideal. We denote by ^ the Jacobson radical adic completion over a semi-local ring. For a ring R, $Q(R)$ denotes the total quotient ring of R and we define $\dim_R 0$ to be $-\infty$ and height R to be $+\infty$.

First we recall the definition of the canonical module.

Definition 0.1 ([6, Definition 5.6]). Let R be an n-dimensional local ring with maximal ideal $\mathfrak{n}$. An R-module C is called *the canonical module* of R if $C\otimes_R \hat{R}\cong\mathrm{Hom}_R(H_{\mathfrak{n}}^n(R), E_R(R/\mathfrak{n}))$.

When R is complete, the canonical module C of R exists and is the module which represents the functor $\mathrm{Hom}_R(H_{\mathfrak{n}}^n(\ \ ), E_R(R/\mathfrak{n}))$, that is, $\mathrm{Hom}_R(H_{\mathfrak{n}}^n(M), E_R(R/\mathfrak{n}))\cong\mathrm{Hom}_R(M, C)$ (functorial) for any R-module M ([6, Satz 5.2]). For elementary properties of the canonical module, we refer the reader to [5, § 6], [6, 5 und 6 Vorträge] and [2, § 1]. If R is a homomorphic image of a Gorenstein ring, R has the canonical module C and it is well known that $C_{\mathfrak{p}}$ is the canonical module of $R_{\mathfrak{p}}$ for every $\mathfrak{p}$ in $\mathrm{Supp}_R(C)$ ([6, Korollar 5.25]). On the other hand, as was shown by Ogoma [7, § 6], there exists a local ring with canonical module and non-Gorenstein formal fibre, hence not a homomorphic image of a Gorenstein ring. But the following fact holds in general and our consideration largely depends on it.

Acknowledgement. Both authors were partially supported by Grant-in-Aid for Co-operative Research.