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Decompositions of tensor products of infinite and finite dimensional representations of semisimple groups

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Introduction. Let $\mathfrak{g}$ be a semisimple Lie algebra over the complex number field $\boldsymbol{C}$. It is interesting to study tensor products of irreducible representations of $\mathfrak{g}$ with a finite dimensional one. For instance, taking the highest or lowest component of the tensor product, we can deduce some properties of an irreducible representation with "singular" parameters from properties of irreducible representations with "regular" parameters.

In 1970's, I.N. Bernstein, I.M. Gel'fand and S.I. Gel'fand [2] used this idea to get a property of Verma modules with singular highest weights from that of Verma modules with regular highest weights. After their work, G. Zuckerman [15] studied this method from functorial point of view and applied it to get the properties of limits of discrete series representations from those of discrete series representations. The method is also used in various fields of representation theory such as the classification of representations [12], the theory of Verma modules [3], [1] and so on.

In this paper, after the method of [15], we try to decompose tensor products of irreducible representations of a connected semisimple Lie group G with a finite dimensional representation F. We hope to apply the results of this paper to ireducible admissible representations of a real reductive group through Langlands' parametrization [13]. So, we are especially interested in the case of discrete series representations. From this point of view, it is interesting that $F\otimes$(discrete series representation) can contain principal series representations, which are induced from a smaller parabolic subgroup (this is one of the results in § 9).

In the first part (§§ 1-4) of this paper, we study the tensor product in general and get fairly natural results. There are two main results in this part. The first one is Proposition 3.3 which says that the character of $F\otimes$(discrete series representation) is a sum of discrete series' character on a compact Cartan subgroup. The second is Proposition 4.3 which says that $F\otimes$(principal series representation) decomposes into (not necessarily irreducible) principal series representations on the whole group G.