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# Approximate solutions for the Cauchy problem for a semilinear hyperbolic system

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## 1. Introduction.

We consider the following system of semi-linear partial differential equations

$$\frac{\partial u}{\partial t}-\lambda\frac{\partial u}{\partial x}=au-uv,$$

$$\frac{\partial v}{\partial t}-\mu\frac{\partial v}{\partial x}=bv+uv,\quad t>0,\quad -\infty<x<\infty,$$

with the initial data $u(x, 0)=\phi(x)$ and $v(x, 0)=\psi(x)$. When $a>0$ and $b<0$ the above system has been considered in ([7]) as a model for the development in time of a prey $u(x, t)$ and predator $v(x, t)$ running on a straight line with speeds $\lambda$ and $\mu$ respectively. The constants $a$ and $b$ are considered as rates of natural multiplication of prey without predator and rate of natural extinction of predator without prey respectively.

By setting $\frac{b}{a}=\gamma$ and $a=\varepsilon$ the above system can be rewritten as

$$\text{(1.1)}\qquad \begin{aligned}\frac{\partial u}{\partial t}-\lambda\frac{\partial u}{\partial x}&=\varepsilon u-uv,\\ \frac{\partial v}{\partial t}-\mu\frac{\partial v}{\partial x}&=\gamma\varepsilon v+uv\end{aligned}$$

and it is in this form that we will consider the problem throughout the paper.

In earlier papers [5, 6] representations for the exact solutions of (1.1) when $\varepsilon=0$ were obtained. Motivated by these, in [7] the authors consider problem (1.1) as a perturbation of the problem when $\varepsilon=0$. Assuming $u_0(x, t)$ and $v_0(x, t)$ to be the exact solutions when $\varepsilon=0$, problem (1.1) is then studied by a perturbation procedure and a solution is sought in the form $\left(\sum_0^\infty u_n(x, t)\varepsilon^n, \sum_0^\infty v_n(x, t)\varepsilon^n\right)$. The main theorem of their paper derives sufficient conditions on $\varepsilon$ in terms of $T$ in order that solutions of the above form exist over $(-\infty, \infty)\times[0, T]$. Uniform convergence of the series is also discussed.

In this paper we study problem (1.1) by following a "Peano-Arzela" type constructive approximation scheme in the spirit of [2]. This approach enables us to obtain global existence results for (1.1) and since uniqueness holds for the