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A theorem on the cohomology of groups and some arithmetical applications

By

Shin-ichi KATAYAMA

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Introduction.

Various cohomology groups related to class field theory have been investigated by many authors. Especially, there are important results on the Galois cohomology groups of ideles and idele classes of finite Galois extensions of algebraic number fields (see, for example [3]). The latter result was first obtained by J. Tate [9]. He also announced the corresponding result for the multiplicative group of the algebraic number field itself in [10], of which the proof was published later in [11], under a more general setting. Recently, we have investigated in [4] the Galois cohomology groups of the factor group of the idele class group by its connected component of the unity. In [5], we have constructed an isomorphism between the Galois cohomology groups of the unit group of a local field and those of some Artin's splitting module.

In this paper, we shall prove the following theorem on the cohomology groups of finite groups and show the known results cited above appear as its special cases.

Let G be a finite group. Suppose that we are given the following commutative diagram of G-modules with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A_1 & \longrightarrow & A_2 & \longrightarrow & A_3 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & B_1 & \longrightarrow & B_2 & \longrightarrow & B_3 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & C_1 & \longrightarrow & C_2 & \longrightarrow & C_3 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & & .
\end{array}$$

Then we have the following theorem

Theorem (A). *With the notation as above, we have*