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# Boundary value problems for second order equations of variable type in a half space

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## § 1. Introduction and statements of results.

This paper is concerned with boundary value problems for homogeneous second order equations of the following type in a half space:

$$(P)\qquad \begin{cases} \dfrac{\partial^2 u}{\partial x^2}+\displaystyle\sum_{j=1}^{n-1}\dfrac{\partial^2 u}{\partial y_j^2}+q(x)\dfrac{\partial^2 u}{\partial t^2}=0\,, & (x,\ y,\ t)\in(0,\ \infty)\times R^{n-1}\times R\,, \\ \lim\limits_{x\to 0} u(x,\ y,\ t)=g(y,\ t) \ \text{ and } \ \lim\limits_{x\to\infty} u(x,\ y,\ t)=0\,, & (y,\ t)\in R^{n-1}\times R\,, \end{cases}$$

where $n=1, 2, 3, \cdots$. The coefficient $q(x)$ satisfies

$(C_0)$ $q(x)$ is real valued bounded and piecewise continnuous in $(0, \infty)$, which we assume throughout this paper. Remark that $q(x)$ may change its sign. We assume one of the following conditions:

$$(C_+)\qquad \varliminf_{x\to\infty} q(x)>0\,,$$

$$(C_-)\qquad \varlimsup_{x\to\infty} q(x)<0\,.$$

As for equations of mixed type, local boundary value problems such as Tricomi problems and Frankl problems were investigated intensively (cf. [1], [4] and [8]). Here we treat with global problems stated as $(P)$ and obtain the integral representation of solutions such as Poisson formula, (see Example 1). This paper continues from [6] and [7]. Being different from problems for elliptic or hyperbolic equations of definite type, the method of localized energy estimates is not effective for our problem $(P)$. Then what we can rely upon? Relating to this question we can clarify our purpose and method below. We see that $(P)$ has at least the linear property: Let $u_j$ be a solution of $(P)$ for $g=e_j$, then $\Sigma\tilde{g}_j u_j$ is a solution of $(P)$ for $g=\Sigma\tilde{g}_j e_j$, where $\tilde{g}_j$ is constant. Here $\Sigma$ can be replaced by the integral symbol $\int$ with respect to some parameters. For example suppose $g(t)=\int_\Gamma e(t,\ \tau)\tilde{g}(\tau)d\tau$ in the case $n=1$, where $e(t,\ \tau)$ is a non vanishing function depending continuously on $\tau$. Then the solution is given by $u(x,\ t)=\int_\Gamma e(t,\ \tau)E(x,\ t,\ \tau)\tilde{g}(\tau)d\tau$, where $e(t,\ \tau)E(x,\ t,\ \tau)$ is a solution of $(P)$ for $g=e(t,\ \tau)$. Here