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A remark on the corona problem for plane domains

Dedicated to Professor Y. Kusunoki on his 60th birthday

By

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1. Introduction.

Let D be a domain in the complex plane, and $H^\infty(D)$ be the Banach algebra of bounded analytic functions on D. We assume that $H^\infty(D)$ contains a nonconstant function. Then, by point evaluations, the domain D can be identified with an open subset of the maximal ideal space $\mathcal{M}(D)$ of $H^\infty(D)$. The corona problem asks whether D is dense in $\mathcal{M}(D)$.

Since L. Carleson [3] solved the corona problem for the unit disk affirmatively, several attempts are made to generalize this result to larger class of plane domains. Among them we are particularly interested in the results of T. Gamelin [6] and M. Behrens [2]. In [6] Gamelin proved the localization principle for $\mathcal{M}(D)$, and by use that he showed some class of plane domains for which the corona problem has an affirmative answer. In the same paper he also introduced some constants $C(D, m, \delta)$ associated with each open set D in $\boldsymbol{C}$, integer $m \geqq 1$, and $\delta > 0$ (see § 2).

In § 2 we show the localization principle concerning the sort of Banach algebras used in the proof of Behrens [2] (Theorem 1). And as its corollary, we know that the Gamelin's constants are finite for the open sets considered in [6] (Theorem 2).

Following W. Deeb [4], we mean by a Δ-domain, a domain obtained from the open unit disk Δ by deleting the origin and a sequence of disjoint closed disks $\Delta_n = \Delta(c_n, r_n) = \{z ; |z - c_n| \leqq r_n\}$ contained in $\Delta \setminus \{0\}$ with c_n tending to 0. In [2] Behrens showed that if the corona problem has a negative answer for some plane domain, then it has a negative answer even for some Δ-domain. Therefore the corona problem for general plane domains is reduced to the case of Δ-domains.

In § 3, using the result of § 2, we shall construct some new examples of Δ-domains for which the corona problem has still an affirmative answer. Actually there is a Δ-domain with $\Sigma |r_n| = +\infty$ for which the corona problem is affirmative.

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