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Stochastic control related to branching diffusion processes

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1. Introduction.

First of all we will recall a controlled stochastic differential equation (CSDE in short) and its Hamilton-Jacobi-Bellman equation (H-J-B eq. in short). [2, 4, 6, 13].

Let Γ be a compact subset of R^k. Let B be a d-dimensional Brownian motion. A Γ-valued process is called an admissible control, if it is progressively measurable with respect to B. $\mathfrak{A}$ denotes the totality of admissible controls.

Consider CSDE for $U\in\mathfrak{A}$,

$$(1.1)\qquad \begin{cases} d\xi(t)=\alpha(\xi(t),\ U(t))dB(t)+\gamma(\xi(t),\ U(t))dt \\ \xi(0)=x. \end{cases}$$

Under the mild conditions, we have a unique solution $\xi=\xi(\cdot,\ x,\ U)$ of (1.1). Define a pay-off function $V(t,\ x,\ \phi,\ U)$ by

$$(1.2)\qquad V(t,\ x,\ \phi,\ U)=E\int_0^t e^{-\int_0^s c(\xi(\theta),U(\theta))d\theta} f(\xi(s),\ U(s))ds + e^{-\int_0^t c(\xi(\theta),U(\theta))d\theta}\phi(\xi(t)),$$

where $\xi(t)=\xi(t,\ x,\ U)$. We want to maximize its value by a suitable choice of $U\in\mathfrak{A}$.

$$(1.3)\qquad V(t,\ x,\ \phi)=\sup_{U\in\mathfrak{A}} V(t,\ x,\ \phi,\ U)$$

is called a value function.

The operator $V(t)$ defined by

$$(1.4)\qquad V(t)\phi(x)=V(t,\ x,\ \phi)$$

becomes a semigroup on a Banach lattice of BUC(R^d)(=totality of bounded and uniformly continuous functions on R^d). Its generatory $\mathfrak{G}$ is given by

$$(1.5)\qquad \mathfrak{G}\phi=\sup_{u\in\Gamma}(A(u)\phi-c(x,\ u)\phi+f(x,\ u)),\qquad \text{for smooth } \phi,$$