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On P. J. Myrberg's approximation theorem for some Kleinian groups

Dedicated to Prof. Y. Kusunoki on his sixtieth birthday.

By

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Introduction.

The apprximation theorem about Fuchsian groups or about the geodesic flow on surfaces of constant negative curvature, obtained by P. J. Myrberg, is based only on some topological and (hyperbolic) geometrical facts. So its proof may be considered elementary.

If we try to extend Myrberg's result to a Kleinian group, we find his method works also efficient for purely loxodromic groups, but we shall face some difficulties for groups which contain parabolic transformations. Such difficulties can be overcome actually by ergodic method.

We shall give in this paper, however, an elementary proof, independent of ergodic theorems, of the approximation theorem for Kleinian groups which are geometrically finite and of the first kind. Moreover, by replacing the terms in our proof, we obtain another proof of the original theorem for Fuchian groups. It seems to the author interesting to find in this paper that parabolic elements, which are generally considered as troublesome existence, turn out to play an important role in the proof.

Further we shall show an analogy to the approximation theorem for classical Schottky groups.

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§1. Preliminaires.

1.1. An isometry on the hyperbolic or non-euclidean 3-space $\boldsymbol{B}^3=\{x=(x_1, x_2, x_3)\in\boldsymbol{R}^3;\ |x|<1\}$, with the Poincaré metric $ds=2|dx|/(1-|x|^2)$, is called a hyperbolic motion.

It is well known that a hyperbolic motion extends to a Möbius transformation on $\hat{\boldsymbol{R}}^3=\boldsymbol{R}^3\cup\{\infty\}$, which has at least one fixed point in $\hat{\boldsymbol{R}}^3$. For simplicity we consider here only orientation preserving motions. Then they are classified into three types: