J. Math. Kyoto Univ. (JMKYAZ)
25-4 (1985) 807-816

On discontinuous subgroups with parabolic transformations of the Möbius groups

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

By

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(Received October 22, 1984)

Introduction

In the theory of Kleinian groups, the following result by H. Shimizu [10] (see also Kra [7, p. 68], Leutbecher [8, Lemma 2.1]) is one of the most fundamental fact.

Proposition. *Let G be a discrete subgroup of $PSL(2; \boldsymbol{C})=SL(2; \boldsymbol{C})/\{\pm E\}$, where E is the unit matrix. If $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in G$, then for any $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G$ we have $|c| \geq 1$ or $c=0$.*

It is known that one can regard a discrete subgroup of $PSL(2; \boldsymbol{C})$ as a group of Möbius transformations, acting discontinuously on the upper half 3-space $\boldsymbol{H}^3=\{(x_1, x_2, x_3) \in \boldsymbol{R}^3;\ x_3>0\}$, and that an element $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2; \boldsymbol{C})$, $c \neq 0$, corresponds to a Möbius transformation with isometric sphere of radius $|c|^{-1}$. In this point of view the above Proposition informs us that if a group G of Möbius transformations, acting discontinuously on $\boldsymbol{H}^3$, contains the parabolic transformation: $(x_1, x_2, x_3) \mapsto (x_1+1, x_2, x_3)$, then for any $T \in G$, $T(\infty) \neq \infty$, the radius of the isometric sphere of T does not exceed one. Our aim is to study this property in the higher dimensional cases. In §2 we shall state the main results in this paper, after providing some definitions and basic facts in §1. The proofs of our results will appear in §§2 and 3.

The author wishes to express his deepest gratitude to Professor Y. Kusunoki for his genial encouragement and valuable comments. And the author also thanks to Dr. M. Taniguchi for his advices.

§1. Preliminaries.

Let $\boldsymbol{R}^n$ and $\overline{\boldsymbol{R}^n}=\boldsymbol{R}^n \cup \{\infty\}$ be the n-dimensional Euclidean space and its one-