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Extreme Pick-Nevanlinna interpolating functions

Dedicated to Professor Yukio Kusunoki on the occasion of his sixtieth birthday

By

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1. In this paper we treat an aspect of Pick-Nevanlinna interpolation theory [13–17] which finds its setting in the theory of convex sets. Specifically, we consider the class I of analytic functions f on Δ, the open unit disk in $\boldsymbol{C}$, which satisfy the following conditions: (i) $\mathrm{Re} f>0$, (ii) $f(0)=1$, (iii) $f(z_k)=w_k$, $k=1,\dots, n$, where the z_k are given distinct points of $\Delta-\{0\}$ and the w_k are given points of $\{\mathrm{Re}\, z>0\}$, $k=1,\dots, n$, n a nonnegative integer. In other words, we are concerned with a harmlessly normalized version of the finite Pick-Nevanlinna interpolation problem where the value 1 is assigned to 0. [For the sake of simplicity of exposition we confine our attention to 0 order interpolation. To be sure, the results obtained will be seen to extend readily.] We suppose that *the class I contains more than one member*. The class I is a compact convex subset of the space of analytic functions on Δ. We seek *to characterize the extreme points of I*, i.e. the members of I *not* admitting a representation of the form $(1-t)f_1+tf_2$, where f_1 and f_2 are distinct members of I and $0<t<1$. It is to be noted that the extreme points associated with a non-normalized finite Pick-Nevalinna problem correspond directly to those associated with a simply related normalized problem as we see with the aid of the map $f\mapsto Af\circ\alpha+iB$, $A>0$, $B\in \boldsymbol{R}$, α a conformal automorphism of Δ. The map in question is a bijection of the space of analytic functions on Δ with positive real part onto itself.

We have the following theorem.

Theorem 1. *The extreme points of I are precisely the members of I having constant valence on* $\{\mathrm{Re}\, z>0\}$, *the value v of the valence satisfying* $1+n\leqq v\leqq 1+2n$.

The proof of the theorem (§2) will be based on the Poisson-Stieltjes representation for analytic functions on Δ with non-negative real part [10, 18] and an elementary fact from Pick-Nevanlinna interpolation theory.

In §3 the extreme points of I will be given a simple representation based on a Nevanlinna representation for the members of I. As a consequence, the extreme points of I will be given a parametric representation the domain of which is the frontier of a convex body in $\boldsymbol{C}^{n+1}$ specified in the manner of the Carathéodory