J. Math. Kyoto Univ. (JMKYAZ)
25-4 (1985) 727–743

# The Cauchy problem for effectively hyperbolic equations (general cases)

Dedicated to Professor Sigeru Mizohata
on the occasion of his sixtieth birthday

By

Nobuhisa IWASAKI

(Communicated by Prof. Mizohata September 20, 1984)

## §0. Introduction

In two previous papers by the author [4] and [5], it was shown for second order operators by the factorization method that effectively hyperbolic operators are strongly hyperbolic. This fact will easily be extended to the general cases of single operators of higher order, because the essential and difficult points of the proofs have been settled through the above papers. This paper aims the proof of this extension by the method which we call the recombination of characteristics. And also it will be shown that it is able to apply to the Cauchy problem for non-linear equations by virtue of the Nash–Moser implicit function theorem. The conclusion of our clarifications and of the results by V. Ya Ivriĭ and V. Petkov [3] is that the effective hyperbolicity is equivalent to the strong hyperbolicity with respect to the non characteristic Cauchy problem for single partial differential equations. We leave the commentary of the related fields to V. Ya Ivriĭ and V. Petkov [3] and to the author [6].

## §1. Notations, Assumptions and Results

We consider a single partial differential operator $p$ of order $m$ on an open set $\Omega$ of $\boldsymbol{R}^{n+1}$. The non characteristic Cauchy problem is to find a solution of the equation $pu=f$ on $\Omega$ satisfying the initial data on a hypersurface of the derivatives up to $m-1$ of $u$ to the conormal direction of the surface. We here use the notation "well posed" defined by following in this connection.

**Definition 1.1.** 1) The Cauchy problem for $p$ is said to be well posed at a point $x^{\sim}$ with respect to a non characteristic direction $\theta \neq 0$ if there exists a neighborhood $\Omega$ of $x^{\sim}$ for any infinitely differentiable function $\phi$ satisfying $\phi(x^{\sim})=0$ and $d\phi(x^{\sim})=\theta$ such that the following statements $(E)_t$ and $(U)_t$ hold for any small $t$.