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On a characterization of the Sobolev spaces over an abstract Wiener space

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Introduction

Let us recall the classical Sobolev spaces in a finite-dimensional case. Consider $H^1(\boldsymbol{R}^d)$ for example. Usually it is defined by means of the Schwartz distribution, that is,

(A) $H^1(\boldsymbol{R}^d) \equiv \{f \in L^2(\boldsymbol{R}^d)$; For each $i=1,\dots, d$, the distribution derivative $\frac{\partial}{\partial x_i} f$ belongs to $L^2(\boldsymbol{R}^d)\}$.

$H^1(\boldsymbol{R}^d)$ is a Hilbert space with norm $\|f\|_{H^1} \equiv \left(\|f\|_{L^2}^2 + \sum_{i=1}^{d} \left\|\frac{\partial}{\partial x_i} f\right\|_{L^2}^2\right)^{1/2}$.

However, if we have to define it without the notion of distribution, we may adopt the following definition.

(B) $H^1(\boldsymbol{R}^d) \equiv$ the completion of the space $C_0^1(\boldsymbol{R}^d)$ with respect to $\| \ \|_{H^1}$.

Or, due to Nikodym, we can take the next one.

(C) $H^1(\boldsymbol{R}^d) \equiv \{f \in L^2(\boldsymbol{R}^d)$; For each $i=1,\dots, d$, there exists a version $\tilde{f}_i$ of f such that $\tilde{f}_i$ is absolutely continuous along almost all lines parallel to the x_i-axis, and its Radon-Nikodym derivative $\frac{\partial}{\partial x_i} \tilde{f}_i$ belongs to $L^2(\boldsymbol{R}^d)$.$\}$

Now, talking about the Sobolev spaces over an abstract Wiener space, two typical definitions are known; one is due to Shigekawa [3] (cf. [5]), and the other is due to Kusuoka-Stroock [2]. In short words, we can say that the former definition is an infinite-dimensional analogue of type (B), and the latter one is that of type (C). In this paper, we first present a theorem in which Shigekawa's Sobolev spaces are characterized, in an analogous way to (A), by means of so-called generalized Wiener functionals. Then, as its application, we will prove that those two definitions of Shigekawa and Kusuoka-Stroock in fact determine the same spaces.